Kingdom of Saudi Arabia
Ministry of Higher Education
Umm Al-Qura University
Faculty of Applied Sciences
Department of Mathematical Sciences

Thermosolutal convection in a horizontal porous layer affected by a vertical magnetic field

Prepared by
Aamal Saeed Ahmed Hatem

Supervised by
Prof. Abdullah Ahmad Abdullah

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شكر وتقدير

(قالوا سبحانك لا علم لنا إلّا ما علمتنا إنك أنت العليم الحكيم) (البقرة/٣٢)

الحمد لله خالق كل شيء منشئ الإنسان من عدم ومعلمه بالقلم، أحمده وأستعينه وأستهديه وأصلي على معلم الأمة وهاديها إلى سواء السبيل سيدنا محمد وعلى آل بيته وصحبه الميامين.

قال عز من قائل (وإذ تأذن ربكم لئن شكرتم لأزيدنكم) (إبراهيم/٧) ومن هنا تتوجه الباحثة بشكرها في المقام الأول إلى الله العلي القدير على توفيقه لها في إنجاز هذا العمل المتواضع راجية منه تعالى السداد والأجر.

ومن بعده سبحانه تودّ الباحثة أن توثق شكرها وتقديرها العميقين لسعادة المشرف على هذا البحث الأستاذ الدكتور/ عبدالله بن أحمد عبدالله والذي كان لها نعم المعين وخير المسدّد، فلم يبخل عليها بوقته وجهده وأغترف لها من معين فكره لإخراج هذا البحث بالصورة الأنيقة فله منها بالغ الدعاء وخالص الثناء راجية من المولى عز وجل أن يبارك فيه ويجزيه كل الخير ويجعل عمله هذا في موازين حسناته.

كما تشكر الباحثة جامعة أم القرى على إتاحة الفرصة لها لإتمام دراستها ولإكمال مسيرة العلم والمساهمة في نهضة هذا الوطن الغالي.

ولا تنسى الباحثة أن تتقدم بالشكر والتقدير لكل من ساهم بتزويدها بالمصادر والمراجع وخاصة القائمين على العمل في مدينة الملك عبدالعزيز للعلوم والتقنية سائلة المولى أن يجزيهم عنها خير الجزاء.

وأخيراً لا يفوت الباحثة أن تتوجه بخالص شكرها وامتنانها لعائلتها المحبة ولكل من آزرها علمياً ومعنوياً من شقيقات ورفيقات داعية إلى الله أن يجعل هذا الدعم في ميزان حسناتهن إنه على كل شيء قدير.

الإهداء

إلى أول من أهدى إلي كتاباً

إلى القدوة التي كانت ولازالت

إلى رمز الحماية والعطاء

والدي

لمن كانت دفء الشتاء وبرد الصيف

لمن علمتني أبجديات الحياة

للنفس الصبورة التي حملتني وتحملتني

أمي

لرفيق الدرب

للروح الطيبة التي عن كل تقصير غضت الطرف

إلى من غرس هذا الأمل وجعله هاجس الليل والنهار

زوجي

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CHAPTER ONE

Introduction

Thermal instability theory has attracted considerable interest and has been recognised as a problem of fundamental importance in many fields of fluid dynamics.The earliest experiments to demonstrate the onset of thermal instability in fluids, are those of Benard (1900,1901). Benard worked with very thin layers of an incompressible viscous fluid, standing on a levelled metallic plate maintained at a constant temperature. The upper surface was usually free and being in contact with the air was at a lower temperature. In his experiments Benard deduced that a certain critical adverse temperature gradient must be exceeded before instability can set in.

Rayleigh (1916) provided a theoretical basis for Benard's experimental results. He showed that the numerical value of the non-dimensional parameter, (known as Rayleigh number)

$$R = \frac{g\alpha\beta}{\kappa\nu} d^4 \tag{1.1}$$

decides whether a layer of fluid heated from below is stable or not, where g is the acceleration of gravity, β is the uniform adverse temperature gradient, d is the depth of the layer and α, ν and κ are the coefficients of volume expansion, kinematic viscosity and thermal diffusivity respectively. Jeffreys (1926,1928),

Low (1929) and Pellew & Southwell (1940) extended Rayleigh's analysis using different boundary conditions. Jeffreys (1930) showed that the onest of thermal instability criterion derived for incompressible fluids could be used for compressible fluids under some certain conditions on the adverse temperature gradient.

Thermal instability theory has been enlarged by the interest in hydrodynamic flows of electrically conducting fluids in the presence of magnetic fields. The presence of such fields in an electrically conducting fluid usually has the effect of inhibiting the development of instabilities. Benard convection in the context of magnetohydrodynamic fluid has been examined by Thompson (1951) and Chandrasekhar (1952) with a linear constitutive relationship between the magnetic field $\boldsymbol{H}$ and the magnetic induction $\boldsymbol{B}$. Chandrasekhar (1952) investigated the Benard problem for the cases of stationary convection and overstability when the magnetic field and gravity act in the same direction for free, rigid and mixed boundaries. He showed that overstability is possible provided $P_m > P_r$ and Q exceeds a critical value where P_m and P_r are respectively the magnetic and viscous Prandtle numbers and the magnitude of Chandrasekhar number Q has the form

$$Q = \frac{\mu^2 H^2 \sigma'}{\rho_0 \nu} d^2 \qquad (1.2)$$

where σ' is the electrical conductivity, μ is the magnetic permeability, ρ_0 is the density of the fluid and H is the magnitude of the magnetic field. Nakagawa (1955,1957) investigated experimentally the problem discussed by Chandrasekhar (1952) for rigid boundaries, and he obtained the critical values of the Rayleigh number for different values of Q with mercury as the electrically conducting liquid. These values coincide with the theoretical values given by Chandrasekhar (1952).

The instability of a layer of fluid heated from below and subjected to Coriolis forces have been studied by Chandrasekhar (1953) and Chandrasekhar & Elbert (1955) for stationary convection and overstability respectively. They showed that the effect of Coriolis forces on the instability of the fluid layer is to inhibit the onest of instability and that the extent of the inhibition depends on the value of the Taylor number T, where

$$T = \frac{4\Omega^2 d^4}{\nu^2} \tag{1.3}$$

and where Ω is the magnitude of the angular velocity. The instability of a layer of fluid heated from below and subjected to the simultaneous action of magnetic field and rotation, has been studied by Chandrasekhar (1954,1956) for the cases of stationary convetion and overstability respectively when the boundaries are free. The results reveal some very unexpected features of the fluid motion under the influence of rotation and magnetic field. The problems discussed by

Chandrasekhar (1954,1956) have been investigated experimentally by Nakagawa (1957) for different layers of mercury. The results obtained by Nakagawa coincide with the theoretical values given by Chandrasekhar. Pearson (1958) discussed Benard's problem when the upper surface was free. He showed that the instability in Benard's problem arises due to surface tension effect rather than buoyancy. Pearson's analysis involved more complicated set of boundary conditions, but a simpler differential equation system, than the corresponding Rayleigh relations. Nield (1964) studied the effects of surface tension and buoyancy in cellular convection of Benard's problem when the lower boundary is a rigid conductor and the upper boundary is free and is subject to a general thermal conditions. He found that surface tension and buoyancy reinforce one another and are tightly coupled. Typical problems in hydrodynamic and hydromagnetic stability have been considered by Chandrasekhar (1981) in his treatise.

Banerjee, et al. (1985) revealed that the analysis of Thompson (1951) and Chandrasekhar (1952) for the overstability case in Benard problem are not correct because the boundary conditions did not include the magnetic boundary conditions. In an attempt to correct this error Banerjee, et al. (1985) showed that the principle of exchange of stabilities is valid if only $QP_r \leq \pi^2$ without the further condition that $P_m > P_r$.

The stability of electrically conducting fluid of variable density in the presence of a horizontal magnetic field and rotation for a rigid boundary has been discussed by Sharma (1988). The convective instability in a horizontal layer of a conductive fluid subject to a horizontal magnetic field and a coupling between buoyancy and surface tension effects for different boundary conditions has been studied by Kaddame (1991). Abdullah and Lindsay (1990,1991) discussed the Benard convection in a non-linear magnetic fluid under the influence of vertical and non-vertical magnetic field for different boundary conditions. They used a non-linear relation between $\boldsymbol{B}$ and $\boldsymbol{H}$ that has been proposed by Roberts (1981) and they showed that this non-linear relationship has no effect on the development of the instabilities through the mechanism of stationary convection but influences the threshold of overstable convection. Abdullah (1992) used Roberts model to discuss the Benard convection when the fluid is subjected to both non-vertical magnetic field and Coriolis forces for different boundary conditions.

Horton & Rogers (1945), Lapwood (1948), Wooding (1960) and Elder (1967) have examined the stability of Benard problem for a fluid in a porous medium using Darcy's law which states that the hydraulic gradient is proprotional to the fluid velocity and to its viscosity, and is inversely proportional to the permeability. The resistance term, calculated from Darcy's law, replaces the usual viscosity term in the equation of motion. A modification of Darcy's law has been

suggested by Brinkman (1947). His model assumes that the viscosity term in the Navier-Stokes equation should be included together with the resistivity term.

There has been considerable interest in the study of the stability of different problems in the presence of porous medium by several workers. Yamamoto and Iwamura (1976) studied the flow with convective acceleration through a porous medium using Brinkman model, Sharma and Thakur (1979) studied the frictional and porosity effects on the Rayleigh –Taylor instability of a composite mixture through porous medium in the presence of a horizontal magnetic field, Rudraiah, et al. (1980) have examined the effects of nonuniform thermal gradient and adiabatic boundaries on convection in porous media for free, rigid and mixed boundaries using Brinkman model, Georgiadis & Catton (1986) studied the Prandtl number effect on Benard convection in porous medium, Kladias and Prasad (1990) studied the thermoconvection instabilities in a horizontal porous layers heated from below when the boundaries are free, Jan (2000) discussed the Benard convection in a horizontal porous layer permeated by a conducting fluid in the presence of vertical magnetic field using Brinkman model. This problem has been extended by Abdullah (2000) for a non-linear magnetic fluid.

Yamamoto & Iwamura (1976) and Rudraiah, et al. (1980) showed that the Brinkman model is valid up to magnitude of $k_1/d^2 \cong 10^{-4}$ or 10^{-3}, where k_1 is the permeability of porous medium.

The effect of the earth's magnetic field on the stability of a layer of porous medium is of interest in geophysics particularly in the study of the earth's core where the earth's mantle, which consists of conducting fluid, behaves like a porous medium which can become convectively unstable as a result of differential diffusion. Another application of the results of flow through a porous medium in the presence of a magnetic field lies in the study of the stability of a convective flow in the geothermal region.

The problem of stability of thermohaline conducting fluid layer in the presence of magnetic field through a porous medium is of importance in geophysics, soil sciences and ground water hydrology. In such problems buoyancy forces can arise not only from density differences due to variations in temperature but also from those due to variations in solute concentrations.

The analysis of such problems has many applications such as the fluid movement in soil, particularly salt water movement in oil reservoirs and waste and fertilizer migration in saturated soil.

Previous studies showed that the presence of salinity in convection problems usually has the effect of inhibiting the development of instabilities when the solute concentration decreases upwards. However it has an opposite effect when the solute concentration increases upwards. Moreover if the fluid is heated from a bove and soluted from below, then the temperature at the bottom of the fluid will be relatively high comparing to the other parts of the fluid. Solar ponds are a good

application of this phenomena and it has been considered in detail by Shirtcliff (1964) and Hoare (1966). Stern (1960) discussed the effect of salinity and temperature in a horizontal layer of water chamber whose top surface is maintained at a higher temperature and salinity than the bottom surface when the boundaries are free. This situation which is taken into account by Stern (1960) exists over most of the ocean ,i.e., salty warm water usually lies above fresher cold water.

The stability of water stratified by both heat and salt is considered by Walin (1964) for the case when the salt concentration increases upwards and the layer is heated from below for stationary and overstability cases. Veronis (1965) studied a layer of fluid that is stably stratified by salt and is heated from below for stationary and overstability cases, when the boundaries are free. This problem can be expected to exist near coastal regions where tidal effects could carry salty warm water in under fresher cold water, or in areas such as the eastern Atlantic where salty mediterranean water flows out through the strait of Gibraltar into the open ocean.

The onset of convection induced by thermal and solute concentration gradients in a horizontal layer of a viscous fluid is studied by Nield (1967) when the layer is heated from below and the solute concentration increases upwards for general set of boundary conditions. Both oscillatory and monotonic instabilities were considered. Nield (1968) studied the thermohaline convection in a porous

medium for a general set of boundary conditions when the fluid is heated and soluted from below. He showed that oscillatory instability may be possible when a strongly stabilizing solute gradient is opposed by a destabilizing thermal gradient. Taunton and lightfoot (1972) extended the work of Nield (1968) to include the instability of both stationary and overstability cases. Sharma & Sharma (1980) studied the thermohaline convection in a layer of conducting fluid heated and soluted from below through porous medium in the presence of a uniform vertical magnetic field. They showed that salinity gradient and magnetic field have stabilizing effect on the system but the medium permeability has a destabilizing effect on the system. A nonlinear thermohaline convection in a rotating porous medium has been studied by Chakrabarti & Gupta (1981).

Chen & Chen (1988) studied the hydrodynamic problem of the onset of finger convection in a porous layer of constant porosity underlying a fluid layer. Patil, et al. (1989) studied the thermohaline instability in a rotating anisotropic porous medium. Sharma, et al. (1990) studied the thermosolutal instability in porous medium of a layer of compressible fluid heated from below and subjected to a stable solute gradient in the absence and presence separately, of rotation and magnetic field when both boundaries are free. They showed that the compressibility, magnetic field and rotation have stabilizing effects.

The thermosolutal convection in a fluid layers heated and soluted from below in a porous medium has been studied by Sharma and Kumari (1992) to include,

separately, the effects of a uniform horizontal magnetic field and a uniform rotation when the boundaries are free. They showed that the stable solute gradient, magnetic field and rotation have stabilizing effects whereas medium permeability has both stabilizing and destabilizing effects on the system. Khare and Sahai (1992) studied the thermosolutal convection in a heterogeneous fluid layer heated and soluted from below in porous medium when the boundaries are free using Brinkman – Oberbeck – Boussinesq equations for the cases of stationary and overstability convection. The effects of various parameters involved were discussed. This problem has been extended by Khare and Sahai (1993) to include the effect of magnetic field and by Khare and Sahai (1994) to include the combined effects of rotation and magnetic field. They showed that Taylor number, Chandrasekhar number and solute Rayleigh number have stabilizing influence on the system. Murty, et al. (1994) used finite element method to study numericaly the stability of thermohaline convection in a rectangular box containing a porous medium using Darcy – Brinkman model.

This thesis studies thermosolutal convective instabilities in a horizontal porous layer permeated by a conducting magnetohydrodynamic fluid using Brinkman model. Stationary convection and overstability cases are considered for the case when the magnetic field is in the vertical direction for different boundary conditions. Analytical solutions were obtained when both boundaries are free and numerical results were presented for the cases of free and rigid boundaries.

The numerical computations were performed using series expansions of Chebyshev polynomials. This method is better suited to the solution of hydrodynamic stability problems than expansions in other sets of orthogonal polynomials. It was first introduced by Lanczos (1938) and Clenshaw (1957) to obtain numerical solutions of linear differential equations. This method has been developed and extensively applied to ordinary differential equations by Fox (1962), Fox and Parker (1968) and Orszag (1961,1971). Orszag (1971) used Chebyshev polynomials to solve the Orr-Sommerfeld equation. His computational treatment of this equation using Chebyshev polynomials established the prominence and viability of this method. Solutions to the primary two-point boundary value problem within a spectral Galerkin formulation have been obtained by Davies, et al. (1988) using Chebyshev polynomials. The expansions in terms of beam functions and Chebyshev polynomials are used to compute solutions to the primary two-point boundary value problem within a spectral collocation formulation by Karageorghis, et al. (1988). Chebyshev solution of laminar boundary layer flow has been studied by Hassanien & El-Hawary (1989). Hassanien, et al. (1996) studied axisymmetric stagnation flow on a cylinder using series expansions of Chebyshev polynomials. This method will be described in detail in chapter two.

CHAPTER TWO

Chebyshev method for boundary value problems

The expansions of Chebyshev polynomials have been used to obtain solutions of many hydrodynamic stability problems. This type of polynomials are better suited to solve hydrodynamic stability problems than other sets of orthogonal polynomials and results of high accuracy are obtained using Chebyshev approximations. In this chapter we shall present some properties of these polynomials together with an example of solving a stability problem using this method.

2.1 Definition:

Let $T_n(x)$ denotes the nth Chebyshev polynomial of the first kind, defined in the interval [-1, 1] by

$$T_n(\cos\theta) = \cos(n\theta) \ , \qquad \theta = cos^{-1}(x) \ , \qquad n \geq 0. \tag{2.1.1}$$

Clearly, from the definition (2.1.1) we can see that

$$|T_n(x)| \leq 1, \qquad |x| \leq 1. \tag{2.1.2}$$

From the trigonometric identity

$$\cos(n+m)\theta + \cos(n-m)\theta = 2\cos(n\theta)\cos(m\theta).$$

We obtain

$$T_{n+m}(x) + T_{|n-m|}(x) = 2T_n(x)T_m(x), \tag{2.1.3}$$

and when $m = 1$, we obtain

$$T_{n+1}(x) - 2xT_n(x) + T_{n-1}(x) = 0, \qquad n \geq 1, \tag{2.1.4}$$

which is called the recurrence relation of the Chebyshev polynomials where

$$T_1(x) = x.$$

In terms of the variable x, we may write (2.1.1) as

$$T_n(x) = \cos(n \cos^{-1}(x)) \tag{2.1.5}$$

from which we can show that

$$T_n(\pm 1) = (\pm 1)^n, \qquad n \geq 0. \tag{2.1.6}$$

By differentiating (2.1.1) with respect to θ, we find that

$$T_n'(\cos\theta) = \frac{n\, sin(n\theta)}{sin\, \theta}, \qquad n \geq 0. \tag{2.1.7}$$

Evaluating this expression at $x = \pm 1$, we find that

$$T_n'(\pm 1) = (\pm 1)^{n-1} n^2, \qquad n \geq 0. \tag{2.1.8}$$

The Chebyshev polynomials are orthogonal over [-1, 1] with respect to the weight function $w(x) = (1 - x^2)^{-\frac{1}{2}}$

i.e. $$\int_{-1}^{1} T_r(x)T_s(x)\, w(x)\, dx = C_r \delta_{sr} \tag{2.1.9}$$

where

$$C_r = \frac{\pi}{2}\begin{cases} 2 & ; \quad r = 0 \\ 1 & ; \quad r > 0 \end{cases}$$

and δ_{sr} is the Kronecher delta defined by

$$\delta_{sr} = \begin{cases} 1 & (s = r) \\ 0 & (s \neq r). \end{cases}$$

For $f(x)$ defined in $-1 \le x \le 1$, the Chebyshev series is given by

$$f(x) = \sum_{r=0}^{\infty} a_{r+1} T_r(x) \tag{2.1.10}$$

where

$$a_{r+1} = \frac{1}{C_r} \int_{-1}^{1} f(x)\, T_r(x)\, w(x)\, dx, \qquad r \ge 0.$$

The derivative of the function $f(x)$ expanded in Chebyshev polynomials can be represented formally as

$$f'(x) = \sum_{r=0}^{\infty} a_{r+1} T_r'(x). \tag{2.1.11}$$

where $T_r'(x)$ is a polynomial of degree $(r-1)$. We can write $T_r'(x)$ as an expansion of Chebyshev polynomials (see Abdullah and Lindsay(1991))

i.e. $$T_r'(x) = \sum_{s=0}^{\infty} B_{s+1,r+1} T_s(x) \qquad r \ge 0 \tag{2.1.12}$$

where $\underline{B}$ is the derivative matrix and where

$$B_{s+1,r+1} = 0 \qquad \textit{if} \qquad s \ge r.$$

From equation (2.1.12)

$$\int_{-1}^{1} T_r'(x)\, T_j(x)\, w(x)\, dx = \sum_{s=0}^{\infty} B_{s+1,r+1} \int_{-1}^{1} T_s(x)\, T_j(x)\, w(x)\, dx\,.$$

Using (2.1.9), we obtain

$$\int_{-1}^{1} T_r'(x)\, T_j(x)\, w(x)\, dx = \sum_{s=0}^{\infty} B_{s+1,r+1} \begin{cases} \pi\delta_{js}\,, & j=0 \\ \dfrac{\pi}{2}\delta_{js}\,, & j>0 \end{cases} \tag{2.1.13}$$

thus

$$\begin{aligned} B_{1,r+1} &= \frac{1}{\pi}\int_{-1}^{1} T_r'(x)\, w(x)\, dx & j=0 \\ B_{j+1,r+1} &= \frac{2}{\pi}\int_{-1}^{1} T_r'(x)\, T_j(x)\, w(x)\, dx. & j\ge 1 \end{aligned} \tag{2.1.14}$$

If we suppose that

$$I_r = \frac{1}{\pi}\int_0^{\pi} \frac{\sin(r\theta)}{\sin\theta}\, d\theta = \begin{cases} 0 & \text{if } r \text{ is even,} \\ 1 & \text{if } r \text{ is odd}, \end{cases} \tag{2.1.15}$$

then from (2.1.7) and (2.1.14), we obtain

$$B_{1,r+1} = \frac{r}{\pi}\int_0^{\pi} \frac{\sin(r\theta)}{\sin\theta}\, d\theta = rI_r. \tag{2.1.16}$$

Also

$$B_{j+1,r+1} = \frac{r}{\pi}\int_0^{\pi} \frac{2\sin(r\theta)\cos(j\theta)}{\sin\theta}\, d\theta$$

$$= \frac{r}{\pi}\left[\int_0^{\pi} \frac{\sin(r+j)\theta}{\sin\theta}\, d\theta + \int_0^{\pi} \frac{\sin(r-j)\theta}{\sin\theta}\, d\theta \right] = r\left(I_{r+j} + I_{r-j}\right). \tag{2.1.17}$$

Hence from equation (2.1.15), we obtain

$$B_{nk}=\begin{cases}0 & \text{if } (n+k) \text{ is even,}\\ (k-1)(2-\delta_{1r}) & \text{if } (n+k) \text{ is odd.}\end{cases} \qquad (2.1.18)$$

where

$$n=j+1, \qquad k=r+1.$$

From (2.1.11) and (2.1.12), we have

$$f'(x)=\sum_{r=0}^{\infty}\sum_{s=0}^{\infty}B_{s+1,r+1}\, a_{r+1}\, T_s(x)$$

and

$$f^{r}(x)=\sum_{r=0}^{\infty}\sum_{s=0}^{\infty}B^{r}{}_{s+1,r+1}\, a_{r+1}\, T_s(x).$$

2.2 Application

Here we shall use the method of Chebyshev polynomials to determine the eigenvalue σ of the classical Benard problem in the presence of magnetic field when the fluid layer is heated from below. The relative equations are

$$\sigma Lw-\sigma P_m Db=L^2 w-a^2\sqrt{R}\theta-QD^2 w,$$

$$\sigma P_r\theta=L\theta+\sqrt{R}w, \qquad (2.2.1)$$

$$\sigma P_m b=Lb+QDw.$$

Where a is the wave number, R is the Rayleigh number, Q is the Chandrasekhar number, $L = D^2 - a^2$, $D = \frac{\partial}{\partial z}$ and w, b and θ are the third components of velocity, magnetic field and temperature. To obtain accurate results and exclude any numerical divergence it is convenient to reduce the order of higher order terms, so let us reduce the order of the term $L^2 w$ in equation $(2.2.1)_1$. If we suppose that

$$\phi = Lw$$

then equations (2.2.1) become

$$\begin{aligned}
&\sigma\phi - \sigma P_m Db = L\phi - a^2\sqrt{R}\theta - QD^2 w,\\
&\sigma P_r\theta = L\theta + \sqrt{R}w,\\
&\sigma P_m b = Lb + QDw,\\
&\sigma \times 0 = Lw - \phi.
\end{aligned} \tag{2.2.2}$$

The system of equations (2.2.2) can be solved using free or rigid boundaries. For free boundaries the conditions are

$$w = \theta = \phi = Db = 0, \qquad 0 \le z \le 1 \tag{2.2.3}$$

and for rigid boundaries the conditions are

$$w = \theta = b = Dw = 0, \qquad 0 \le z \le 1. \tag{2.2.4}$$

Since the Chebyshev polynomials are defined in the interval [-1,1] and our problem is defined in the interval [0,1], then we may introduce the variable x such that

$$x = 2z - 1,$$

$$D(\ ) = 2\frac{d(\ )}{dx} = 2B(\ ),$$

$$D^2(\ ) = 4\frac{d^2(\ )}{dx^2} = 4B^2(\ ),$$

$$L(\ ) = (D^2 - a^2)(\ ) = 4\frac{d^2(\ )}{dx^2} - a^2(\ ) = 4\left[\frac{d^2(\ )}{dx^2} - \left(\frac{a}{2}\right)^2(\ )\right] = 4V(\ ),$$

where $V(\ ) = \frac{d^2(\ )}{dx^2} - \left(\frac{a}{2}\right)^2(\ )$ and B is the derivative matrix defined in (2.1.18).

Now we express all the variables of the problem in terms of Chebyshev polynomials in the following way

$$(w, \theta, \phi, b) = \sum_{n=0}^{\infty}(a_{n+1}, b_{n+1}, C_{n+1}, d_{n+1})T_n(x). \tag{2.2.5}$$

Apply (2.2.5) into the governing equations (2.2.2) and the boundary conditions (2.2.3) and (2.2.4), then the governing equations become

$$\sigma\left(C_{n+1}-2P_mBd_{n+1}\right)=4VC_{n+1}-a^2\sqrt{R}b_{n+1}-4QB^2a_{n+1},$$

$$\sigma P_rb_{n+1}=4Vb_{n+1}+\sqrt{R}a_{n+1},$$

$$\sigma P_md_{n+1}=4Vd_{n+1}+2QBa_{n+1}, \tag{2.2.6}$$

$$\sigma\times 0=4Va_{n+1}-C_{n+1},$$

the free boundary conditions become

$$\sum_{n=0}^{\infty}a_{n+1}T_n(x)=\sum_{n=0}^{\infty}b_{n+1}T_n(x)=\sum_{n=0}^{\infty}C_{n+1}T_n(x)=\sum_{n=0}^{\infty}d_{n+1}T_n'(x)=0,\quad -1\le x\le 1, \tag{2.2.7}$$

and the rigid boundary conditions become

$$\sum_{n=0}^{\infty}a_{n+1}T_n(x)=\sum_{n=0}^{\infty}b_{n+1}T_n(x)=\sum_{n=0}^{\infty}d_{n+1}T_n(x)=\sum_{n=0}^{\infty}a_{n+1}T_n'(x)=0,\quad -1\le x\le 1. \tag{2.2.8}$$

Equations (2.2.6) can be written in the form

$$\sigma\,\underline{E}\,\underline{X}=\underline{F}\,\underline{X}$$

where

$$\underline{E}=\begin{bmatrix} I & 0 & -2P_mB & 0\\ 0 & P_rI & 0 & 0\\ 0 & 0 & P_mI & 0\\ 0 & 0 & 0 & 0\end{bmatrix},\qquad \underline{F}=\begin{bmatrix} 4V & -a^2\sqrt{R}I & 0 & -4QB^2\\ 0 & 4V & 0 & \sqrt{R}I\\ 0 & 0 & 4V & 2QB\\ -I & 0 & 0 & 4V\end{bmatrix}$$

and

$$\underline{X}=\begin{bmatrix}C_{n+1} & b_{n+1} & d_{n+1} & a_{n+1}\end{bmatrix}^T.$$

Using (2.1.6) and (2.1.8), the free boundary conditions (2.2.7) become

$$\sum_{n=0}^{\infty}(\pm 1)^n a_{n+1} = \sum_{n=0}^{\infty}(\pm 1)^n b_{n+1} = \sum_{n=0}^{\infty}(\pm 1)^n C_{n+1} = \sum_{n=0}^{\infty}(\pm 1)^{n-1} n^2 d_{n+1} = 0 \qquad (2.2.9)$$

and the rigid boundary conditions (2.2.8) become

$$\sum_{n=0}^{\infty}(\pm 1)^n a_{n+1} = \sum_{n=0}^{\infty}(\pm 1)^n b_{n+1} = \sum_{n=0}^{\infty}(\pm 1)^n d_{n+1} = \sum_{n=0}^{\infty}(\pm 1)^{n-1} n^2 a_{n+1} = 0. \qquad (2.2.10)$$

Notice that each element of the matrix $\underline{E}$ and $\underline{F}$ represents a square matrix. The order of this matrix depends on the number of Chebyshev polynomials used for the required accuracy. In general the boundary conditions (2.2.9) or (2.2.10) should be inserted in the first and second rows of each diagonal element of the matrix $\underline{F}$. The corresponding rows in the other elements of $\underline{F}$ and $\underline{E}$ must be set to zero.

Each diagonal element of $\underline{E}$ and $\underline{F}$ corresponds to a variable element in $\underline{X}$. So in this problem we can see from equations (2.2.6) that the first, the second, the third and the fourth diagonal elements correspond to ϕ, θ, b and w respectively. Thus the first row of each diagonal element of $\underline{E}$ contains the condition of the corresponding variable at $x = 1$ whereas the second row contains the condition of that variable at $x = -1$.

If the boundary conditions in (2.2.9) or (2.2.10) are of the form $\sum_{n=0}^{\infty}(\pm 1)^n \alpha_{n+1} = 0$, then the first row of the corresponding matrix is of the form $(1, 1, 1, ...)$ and the second row is of the form $(1, -1, 1, -1, ...)$. However if the

boundary conditions in (2.2.9) or (2.2.10) are of the form $\sum_{n=0}^{\infty}(\pm 1)^{n-1} n^2 a_{n+1} = 0,$ then the first row is of the form $(0,1,4,9,16,....)$ and the second row is of the form $(0,-1,4,-9,16,....)$.

In the case of rigid boundaries, the condition $\phi = 0$ is replaced by $Dw = 0$, so we remove the condition of $\phi = 0$ which is written in the diagonal element of the first row of the matrix $\underline{E}$ and insert the condition $Dw = 0$ in the same row but in the column corresponds to the variable w, i.e. in the fourth column in this problem.

In practice, we cannot use infinite series of Chebyshev polynomials and we have to rely on a finite approximation of suitable accuracy, so we look for an approximation of the form

$$Y(x) = \sum_{n=0}^{N} a_{n+1} T_n(x)$$

where N is the number of Chebyshev polynomials required.

In the theory of convection, we introduce time dependence through $\exp(\sigma t)$ and instability occurs if any eigenvalue σ has a positive real part. The most competitive or " Least stable" eigenvalue has the algebraically largest real part. At this stage the eigenvalue problem (2.2.6) together with the boundary conditions (2.2.9) and (2.2.10) can be solved using the numerical routine F02BJF which is

part of a mathematical library called numerical algorithm group (NAG). The fortran code for this example is listed in Appendix 1.

2.3 The golden section search

Here we shall explain a method used in the following chapter to obtain the minimum of the Rayleigh number as a function of the wave number. It deals with a unimodel function, i.e. a function which has only a single local minimum. This method is an iterative method and it has been termed the golden section search since it depends on a ratio r known to the early Greeks as the golden section ratio and is given by

$$r = \frac{1}{2}\left(\sqrt{5}-1\right) \approx 0.6180339887$$

where $$r^2 = 1 - r.$$

Suppose that $G(x)$ has a single minimum in the interval $[a,b]$. In each step we need to replace this interval by a smaller one that is also contains the minimum point. Moreover in each step we need to specify two values of G at two particular points in $[a,b]$. i.e.

$$x = a + r^2(b-a) \qquad u = G(x),$$

$$y = a + r(b-a) \qquad v = G(y).$$

There are two cases to consider :

Case 1 $(u > v)$:

Since G is unimodal, the minimum of G must be in the interval $[x, b]$ and so in the next step G is evaluated at x^*, y^* where

$$x^* = x + r^2(b - x),$$

$$y^* = x + r(b - x).$$

we note that

$$x^* = a + r^2(b-a) + r^2[b - a - r^2(b-a)]$$

$$= a + (2r^2 - r^4)(b-a)$$

$$= a + \left[1 - (1 - r^2)^2\right](b-a)$$

$$= a + (1 - r^2)(b-a)$$

$$= a + r(b - a)$$

$$= y.$$

Thus

$$G(x^*) = G(y) = v$$

Case 2 $(u \le v)$:

The minimum of G must be in the interval $[a, y]$ and so G is evaluated at x^*, y^* where

$$x^* = a + r^2(y - a),$$

$$y^* = a + r(y - a).$$

We note that

$$y^* = a + r[a + r(b - a) - a]$$

$$= a + r[r(b - a)]$$

$$= a + r^2(b - a)$$

$$= x.$$

Thus

$$G(y^*) = G(x) = u.$$

In both cases the valuse of G at one of the new points is already known. Also we note that

$$y - a = b - x = b - a - r^2(b - a)$$

$$= (b - a)(1 - r^2)$$

$$= r(b - a),$$

so that after N iterations of the method, the interval $[a, b]$ is reduced to one of length $r^N(b - a)$ and so if ε is the required accuracy then we need to choose N such that

$$r^N(b-a)<\varepsilon$$

i.e. $$r^N<\frac{\varepsilon}{b-a}$$

and so $$\log r^N<\log\frac{\varepsilon}{b-a}$$

i.e. $$N>\frac{\log\left(\frac{\varepsilon}{b-a}\right)}{\log r}.$$

CHAPTER THREE

Thermosolutal convection in a horizontal porous layer affected by a vertical magnetic field.

3.1 Mathematical formulation

Consider an infinte horizontal layer occupied by a porous medium permeated by an incompressible, thermally and electrically conducting viscous solutal fluid of density ρ. The fluid is subject to a constant gravitational acceleration in the negative x_3 direction. A constant magnetic field is imposed a cross the layer in the positive x_3 direction and x_1 and x_2 axes are selected from a right –handed system of cartesian coordinates in which the magnetic induction $\boldsymbol{B}$ has the representation $\boldsymbol{B} = (0,0,B)$. (see figure 1)

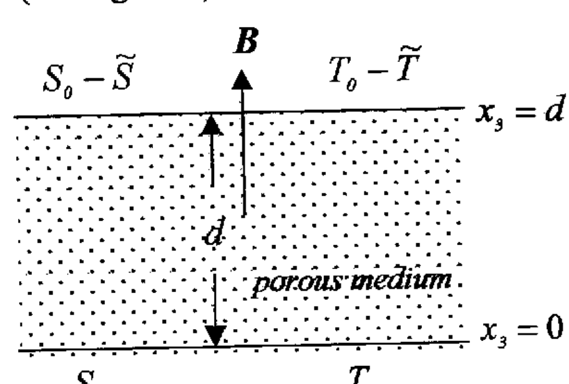


Figure 1

In order to fully describe the nature of this model we need to discuss the interaction between electromagnetic and mechanical effects and so we define

$\boldsymbol{H}, \boldsymbol{E}$ and $\boldsymbol{J}$ to be respectively the magnetic field, the electric field and the current density. The magnetic variables are required to satisfy the Maxwell's equations

$$\begin{aligned}
&\text{div } \boldsymbol{B} = B_{i,i} = 0, \\
&(\text{curl } \boldsymbol{H})_i = e_{ijk} H_{k,j} = J_i, \qquad (3.1.1) \\
&(\text{curl } \boldsymbol{E})_i = e_{ijk} \; E_{k,j} = - \frac{\partial B_i}{\partial t}.
\end{aligned}$$

where the displacement current has been neglected as is customary in this type of problems and where the current density $\boldsymbol{J}$ is given by

$$\eta J_i = E_i + e_{ijk} \; \upsilon_j B_k, \qquad (3.1.2)$$

where υ_j is the fluid velocity and η is the resistivity (assumed constant). The linear relation between $\boldsymbol{B}$ and $\boldsymbol{H}$ has the form

$$B_i = \mu H_i \; ,$$

where μ is the permeability. If we use the conventional notations, namely θ, is absolute temperature, C is solute mass concentration and P is hydrostatic pressure then the relevant equations of motion become

$$\rho \dot{\upsilon}_i = -p_{,i} + 2\rho \, \nu \, d_{ij,j} + \rho g_i + (\boldsymbol{J} \times \boldsymbol{B})_i - \frac{\mu'}{k_1} \upsilon_i, \qquad (3.1.3)$$

where $d_{ij} = \frac{1}{2}(\upsilon_{i,j} + \upsilon_{j,i})$, μ' is the dynamic viscosity, k_I is the permeability of porous medium, ν is the kinematic viscosity and $\dot{\upsilon}_i = \frac{\partial \upsilon_i}{\partial t} + \frac{\partial \upsilon_i}{\partial x_j}\frac{\partial x_j}{\partial t}$. The term $(\boldsymbol{J} \times \boldsymbol{B})_i$ in (3.1.3) can be written as

$$
\begin{aligned}
(\boldsymbol{J} \times \boldsymbol{B})_i &= (\text{curl } \boldsymbol{H} \times \boldsymbol{B})_i \\
&= e_{ijk} \, (\text{curl}\boldsymbol{H})_j \, B_k \\
&= e_{ijk} e_{jrs} \, H_{s,r} B_k \\
&= (\delta_{kr}\delta_{is} - \delta_{ks}\delta_{ir}) H_{s,r} B_k \\
&= H_{i,k} B_k - H_{k,i} B_k \\
&= \frac{1}{\mu}(B_{i,k} B_k - B_{k,i} B_k).
\end{aligned}
$$

Since $B^2 = B_k B_k$, then $B_{,i}^{\,2} = 2\, B_k B_{k,i}$. Thus

$$
(\boldsymbol{J} \times \boldsymbol{B})_i = \frac{1}{\mu} B_{i,k} B_k - \frac{1}{2\mu} B_{,i}^{\,2},
$$

and equation (3.1.3) becomes

$$
\rho \dot{\upsilon}_i = -P_{,i} + 2\rho \nu \, d_{ij,j} + \rho \, g_i + \frac{1}{\mu} B_{i,k} B_k - \frac{\mu'}{k_I} \upsilon_i. \tag{3.1.4}
$$

where $P = p + \frac{1}{2\mu} B^2$. Using equations $(3.1.1)_3$ and (3.1.2) the magnetic induction satisfies the partial differential equation

$$\frac{\partial \boldsymbol{B}}{\partial t} = curl(\upsilon \times \boldsymbol{B}) - \eta \,\mathrm{curl}\, \boldsymbol{J}$$

$$= e_{ijk}\, e_{krs} (\upsilon_r B_s)_{,j} - \eta\, e_{ijk} J_{k,j}$$

$$= (\delta_{ir}\, \delta_{js} - \delta_{is} \delta_{jr}) (\upsilon_{r,j} B_s + \upsilon_r\, B_{s,j}) - \eta e_{ijk} J_{k,j}$$

$$= \upsilon_{i,j} B_j - \upsilon_j B_{i,j} - \eta e_{ijk} J_{k,j}. \tag{3.1.5}$$

We now use the Boussinesq approximation, namely that density is constant every where except in the body force term where the density is linearly propotional to temperature and solute mass concentration, i.e.

$$\rho = \rho_0 \,(1 - \alpha\, \theta + \alpha'\, \mathrm{C}\,),$$

where ρ_0 is the fluid density at temperature T_0 and $\alpha\,, \alpha'$ are the coefficients of volume and solvent expansion. Then the governing field equations are

$$\dot{\upsilon}_i = -(P/\rho_0)_{,i} + \nu \nabla^2 \upsilon_i - g(1 - \alpha\theta + \alpha' C)\delta_{i3} + \frac{1}{\rho_0 \mu} B_{i,k} B_k - \frac{\nu}{k_1} \upsilon_i ,$$

$$\upsilon_{i,i} = 0,$$

$$\dot{\theta} = \kappa \nabla^2 \theta,$$

$$\dot{C} = \kappa' \nabla^2 C \qquad (3.1.6)$$

$$B_{i,i} = 0,$$

$$J_i = e_{ijk} H_{k,j},$$

$$\frac{\partial B_i}{\partial t} = \upsilon_{i,j} B_j - \upsilon_j B_{i,j} - \eta e_{ijk} J_{k,j}.$$

where κ and κ' are the coefficients of thermal and solute diffusivity and $\nu = \dfrac{\mu'}{\rho_0}$.

We may observe that equations (3.1.6) have a steady solution in which

$$\upsilon = 0, \qquad \boldsymbol{J} = 0,$$

$$\theta = \theta(x_3) = T_0 - \beta x_3, \qquad \beta = \frac{\widetilde{T}}{d},$$

$$C = C(x_3) = S_0 - \beta' x_3, \qquad \beta' = \frac{\widetilde{S}}{d}, \qquad (3.1.7)$$

$$P = P(x_3), \quad \boldsymbol{B} = (0,0,B) \;; \quad B = \text{constant}.$$

where β is the uniform adverse temperature gradient and β' is the adverse concentration gradient. The temperatures on the planes $x_3 = 0$ and $x_3 = d$ are T_0 and T_0 - $\widetilde{T}$ respectively and the solute concentrations on the planes $x_3 = 0$ and $x_3 = d$ are S_0 and S_0 - $\widetilde{S}$ respectively. Although we consider the case in which the external magnetic field is normal to the layer of fluid, a similar analysis can be applied to any constant $\boldsymbol{B}$. Let

$$\upsilon = \mathbf{0} + \varepsilon\hat{\upsilon}, \qquad \boldsymbol{J} = \mathbf{0} + \varepsilon\,\hat{\boldsymbol{J}}, \qquad \theta = \theta + \varepsilon\,\hat{\theta},$$

$$C = C + \varepsilon\,\hat{C}, \qquad P = P + \varepsilon\,\hat{P}, \qquad \boldsymbol{B} = (0,0,B) + \varepsilon\,\hat{\boldsymbol{b}}.$$

where $\hat{\upsilon}$, $\hat{\boldsymbol{J}}$, $\hat{\theta}$, $\hat{C}$, $\hat{P}$ and $\hat{\boldsymbol{b}}$ are respectively the linear perturbation of velocity, current density, temperature, solute concentration, pressure and magnetic induction, then we may verify that the linearised version of equations (3.1.6) are

$$\frac{\partial \hat{\upsilon}_i}{\partial t} = -(\hat{P}/\rho_0)_{,i} + \nu \nabla^2 \upsilon_i + g(\alpha \hat{\theta} - \alpha' \hat{C})\delta_{i3} + \frac{1}{\rho_0 \mu} B \hat{b}_{i,3} - \frac{\nu}{k_1} \hat{\upsilon}_i ,$$

$$\hat{\upsilon}_{i,i} = 0 ,$$

$$\frac{\partial \hat{\theta}}{\partial t} - \beta \hat{\upsilon}_3 = \kappa \nabla^2 \theta,$$

$$\frac{\partial \hat{C}}{\partial t} - \beta' \upsilon_3 = \kappa' \nabla^2 \hat{C}, \tag{3.1.8}$$

$$\hat{b}_{i,i} = 0,$$

$$\hat{J}_i = \frac{1}{\mu} e_{ijk} \hat{b}_{k,j},$$

$$\frac{\partial \hat{b}_i}{\partial t} = B \hat{\upsilon}_{i,3} \; - \eta e_{ijk} \hat{J}_{k,j}.$$

At this stage we introduce the dimensionless variables x_i^* , t^* , υ_i^* , J_i^* , C^*, θ^*, P^* and b_i^*, such that

$$x_i = dx_i^* , \quad t = \frac{d^2}{\nu} t^*, \quad \hat{\upsilon}_i = \frac{\kappa}{d} \upsilon_i^*, \quad \hat{J}_i = \frac{\kappa \nu \rho_0}{d^3 B} J_i^*, \quad \hat{C} = \frac{\kappa}{d} \sqrt{\frac{\nu |\beta'|}{\kappa' \alpha' g}} C^*,$$

$$\hat{\theta} = \frac{\kappa}{d} \sqrt{\frac{\nu |\beta|}{\kappa \alpha g}} \theta^*, \quad \hat{P} = \frac{\rho_0 \kappa \nu}{d^2} P^*, \quad \hat{b}_i = \frac{\rho_0 \kappa \nu \mu}{d^2 B} b_i^*.$$

After this non-dimensionalization, equations (3.1.8) become

$$\frac{\partial \upsilon_i}{\partial t} = -P_{,i} + \nabla^2 \upsilon_i + \sqrt{R_t}\,\theta\,\delta_{i3} - \sqrt{R_s}\,C\,\delta_{i3} - \frac{1}{N}\upsilon_i + b_{i,3}\,,$$

$$\upsilon_{i,i} = 0\,,$$

$$P_r \frac{\partial \theta}{\partial t} + H\sqrt{R_t}\,\upsilon_3 = \nabla^2 \theta\,,$$

$$P_r' \frac{\partial C}{\partial t} + H'\sqrt{R_s}\,\upsilon_3 = \nabla^2 C\,, \tag{3.1.9}$$

$$b_{i,i} = 0,$$

$$J_i = e_{ijk} b_{k,j}\,,$$

$$P_m \frac{\partial b_i}{\partial t} = Q\upsilon_{i,3} - e_{ijk} J_{k,j}.$$

where the (*) superscript has been dropped but all the variables are now non-dimensional and where the non-dimensional numbers R_t, R_s, N, P_r, P_r', P_m and Q are given by

$$R_t = \frac{\alpha\, g\,|\beta|}{\kappa\,\nu} d^4\ ,\quad R_s = \frac{\alpha'\, g\,|\beta'|}{\kappa'\,\nu} d^4\,,\quad N = \frac{k_1}{d^2}\ ,\quad P_r = \frac{\nu}{\kappa}\ ,$$

$$P_r' = \frac{\nu}{\kappa'}\ ,\quad P_m = \frac{\mu\,\nu}{\eta}\ ,\quad Q = \frac{B^2 d^2}{\rho_0\,\nu\,\eta}.$$

where R_t is the thermal Rayleigh number, R_s is the solute Rayleigh number, N is the permeability of porous medium, P_r is the viscous Prandtle number, P_r' is

the Schmidt number, P_m is the magnetic Prandtle number, Q is the Chandrasekhar number and where

$$H = -\frac{\beta}{|\beta|} = \begin{cases} 1 & \text{when heating from above}, \\ -1 & \text{when heating from below.} \end{cases}$$

$$H' = -\frac{\beta'}{|\beta'|} = \begin{cases} 1 & \text{when solute concentration increases upwards}, \\ -1 & \text{when solute concentration decreases upwards.} \end{cases}$$

3.2 The boundary conditions

The fluid is confined between the planes $x_3 = 0$ and $x_3 = d$ and on these planes, we need to specify mechanical, thermal, solutal and electromagnetic conditions. Suitable mechanical conditions assume either a free or a rigid boundary; suitable thermal conditions assume either an insulating or a perfectly conducting boundary; suitable solutal conditions assume either a permeable or an impermeable boundary and suitable electromagnetic conditions assume an electrically insulating or a perfectly conducting boundary.

Mechanical conditions

On a stationary rigid boundary, $\upsilon_3 = 0$ on $x_3 = 0, 1$, i.e. no slip occurs and the normal component of velocity must vanish on these surfaces. This implies that each component of the fluid velocity and all of x_1, x_2 partial derivatives of each component of the fluid velocity are zero. On a stationary free boundary $\frac{\partial \upsilon_3}{\partial n} = 0$,

where n is the normal to the cylindrical boundary drawn outwards and since

$\frac{\partial \theta}{\partial n} = \frac{\partial \upsilon_3}{\partial n}$ and $\theta = 0$ then $\upsilon_3 = 0$.

Also the x_3 component of velocity is zero and the absence of shear stress requires all partial x_3 derivatives of the tangential velocity components to be zero.

If we define ξ to be the fluid vorticity then $\xi = curl \upsilon$ and the mechanical boundary conditions can be expressed by the equations,

$$\xi_3 = 0 \qquad \text{on a rigid boundary,}$$
$$\frac{\partial \xi_3}{\partial x_3} = 0 \qquad \text{on a free boundary.} \tag{3.2.1}$$

If the fluid is also incompressible, then υ_3 satisfies the additional property

$$\frac{\partial \upsilon_3}{\partial x_3} = 0 \qquad \text{on a rigid boundary,}$$
$$\frac{\partial^2 \upsilon_3}{\partial x_3^{\ 2}} = 0 \qquad \text{on a free boundary.} \tag{3.2.2}$$

Thermal conditions

For a perfectly conducting boundary the temperatures of the boundary and impinging fluid match whereas on a perfectly insulating boundary, no heat transfer can take place between the fluid and the surroundings hence the normal

derivative of temperature is zero. In mathematical terms, the thermal conditions are

$$\theta = \theta_{ext} \qquad \text{on a conducting boundary,}$$

$$\theta_{,3} = 0 \qquad \text{on an insulating boundary.} \tag{3.2.3}$$

where θ_{ext} is the temperature of the region exterior to the fluid boundary.

Solute conditions

The possible solute conditions are

$$C = C_{ext} \qquad \text{on a permeable boundary,}$$

$$C_{,3} = 0 \qquad \text{on an impermeable boundary.} \tag{3.2.4}$$

where C_{ext} is the solute concentration in the region exterior to the boundary.

Electromagnetic conditions

On a perfectly insulating electromagnetic boundary, no current can flow to the exterior region i.e $J_3 = 0$ and the magnetic field is continous across the boundary with the external magnetic field being derived from a scalar potential since $(\text{curl } \boldsymbol{H})_3 = 0$ in the exterior region. On a stationary perfectly conducting boundary, there is no magnetic field passes through to the exterior region i.e.

$b_3 = 0$ and no surface components of electric field, i.e. $E_1 = E_2 = 0$ on $x_3 = 0,\ d$. Also the surface components of the current density are zero i.e. $J_1 = J_2 = 0$ on $x_3 = 0,\ d$. And since div $\boldsymbol{J} = 0$

then

$$J_{,3} = 0. \tag{3.2.5}$$

3.3 The eigenvalue problem

Now we shall discuss the linear stability of the conducting solution (3.1.7). In order to do that we shall construct the related eigenvalue problem from equations (3.1.9) and the boundary conditions. In many convection problems, the vector components parallel to the direction of gravity (i.e. the x_3 direction) play a central role and so we introduce the variables w, b, J, ξ and z by the definitions

$$w = \upsilon_3, \quad b = b_3, \quad J = J_3, \quad \xi = \xi_3, \quad z = x_3.$$

When we take the curl of equations $(3.1.9)_1$ and $(3.1.9)_7$, we obtain

$$\frac{\partial \xi_i}{\partial t} = \nabla^2 \xi_i + e_{ijk}\sqrt{R_t}\,\theta_{,j}\,\delta_{k3} - e_{ijk}\sqrt{R_s}\,C_{,j}\,\delta_{k3} + J_{i,3} - \frac{1}{N}\xi_i,$$

$$P_m \frac{\partial J_i}{\partial t} = Q\xi_{i,3} + \nabla^2 J_i\,, \tag{3.3.1}$$

and if we take the (curl curl) of equation $(3.1.9)_1$, we obtain

$$\frac{\partial}{\partial t}\nabla^2 v_i = \nabla^4 v_i - \sqrt{R_t}\,(\theta_{,3i} - \nabla^2\theta\,\delta_{i3}) + \sqrt{R_s}(C_{,3i} - \nabla^2 C\,\delta_{i3}) + \nabla^2 b_{i,3} - \frac{1}{N}\nabla^2 v_i. \quad (3.3.2)$$

The third components of equations (3.3.1), (3.3.2) and $(3.1.9)_{3,4,7}$ are

$$\frac{\partial \xi}{\partial t} = \nabla^2 \xi + J_{,3} - \frac{1}{N}\xi,$$

$$P_m \frac{\partial J}{\partial t} = Q\xi_{,3} + \nabla^2 J,$$

$$\frac{\partial}{\partial t}\nabla^2 w = \nabla^4 w + \sqrt{R_t}(\theta_{,11} + \theta_{,22}) - \sqrt{R_s}(C_{,11} + C_{,22}) + \nabla^2 b_{,3} - \frac{1}{N}\nabla^2 w,$$

$$P_m \frac{\partial b}{\partial t} = Q\,w_{,3} + \nabla^2 b, \qquad (3.3.3)$$

$$P_r \frac{\partial \theta}{\partial t} + H\sqrt{R_t}\,w = \nabla^2 \theta,$$

$$P_r' \frac{\partial C}{\partial t} + H'\sqrt{R_s}\,w = \nabla^2 C.$$

Now we look for a solution of the form

$$\Phi = \Phi(z)\ \exp[i(nx + my) + \sigma t\ ]$$

where n,m are the wave numbers of the harmonic distrubance and σ is the growth rate. Thus the relative equations become

$$\sigma\,\xi = L\xi + DJ - \frac{1}{N}\xi,$$

$$\sigma\,P_m J = QD\xi + LJ,$$

$$\sigma\,Lw = L^2 w - a^2\sqrt{R_t}\;\theta + a^2\sqrt{R_s}\;C + L(Db) - \frac{1}{N}Lw,$$

$$\sigma\,P_m b = QDw + Lb, \tag{3.3.4}$$

$$\sigma\,P_r\theta = L\theta - H\sqrt{R_t}\;w,$$

$$\sigma\,P_r' C = LC - H'\sqrt{R_s}\,w.$$

where D is the operator $\frac{\partial}{\partial z}$, $a\left(=\sqrt{n^2+m^2}\right)$ is the wave number and L is the operator (D^2-a^2). We may observe that equations $(3.3.4)_{1,2}$ depend on ξ and J only and decoupled from w, b, θ and C. We may eliminate b, θ, and C from $(3.3.4)_3$ using equations $(3.3.4)_{4,5,6}$ to obtain

$$L(L-\sigma)\;(L-\sigma\,P_m)(L-\sigma\,P_r)(L-\sigma\,P_r')\,w - a^2 H\,R_t\,(L-\sigma\,P_m)(L-\sigma\,P_r')w$$

$$+\,a^2 H' R_s(L-\sigma\,P_m)(L-\sigma\,P_r)w - QL(L-\sigma\,P_r)(L-\sigma\,P_r')D^2 w$$

$$-\frac{1}{N}L(L-\sigma\,P_m)(L-\sigma\,P_r)(L-\sigma\,P_r')w = 0 \tag{3.3.5}$$

which we can put in the form

$$\sigma^4 P_m P_r P_r' Lw - \sigma^3 \left\{ (P_r' P_r + P_m P_r' + P_m P_r + P_m P_r P_r') L^2 w - \frac{1}{N} P_m P_r P_r' Lw \right\}$$

$$+ \sigma^2 \Big\{ (P_r + P_m + P_r' + P_r P_r' + P_m P_r' + P_m P_r) L^3 w - [Q P_r P_r' + \frac{1}{N}(P_r P_r' + P_m P_r' + P_r P_m)] L^2 w$$

$$- Q a^2 P_r P_r' Lw - a^2 P_m (H R_t P_r' - H' R_s P_r) w \Big\} - \sigma \Big\{ (1 + P_r' + P_r + P_m) L^4 w$$

$$- \left[Q(P_r + P_r') + \frac{1}{N}(P_r + P_r' + P_m) \right] L^3 w - Q a^2 (P_r + P_r') L^2 w$$

$$- a^2 [H R_t (P_r' + P_m) - H' R_s (P_r + P_m)] Lw \Big\} + L^5 w - (Q + \frac{1}{N}) L^4 w - Q a^2 L^3 w$$

$$- a^2 (H R_t - H' R_s) L^2 w = 0 \qquad (3.3.6)$$

which is a tenth order ordinary differential equation to be satisfied by w.

3.4 Stability analysis

Here we shall consider the variation of the Rayleigh number as a function of the wave number a, the solute Rayleigh number R_s, the Chandrasekhar number Q and the permeability of porous medium N for the stationary convection case. i.e. when $\sigma = 0$. We deduce from (3.3.4) that

$$L^2 w - a^2 \sqrt{R_t}\theta + a^2 \sqrt{R_s} C + L(Db) - \frac{1}{N} Lw = 0,$$

$$Lb + QDw = 0,$$

$$L\theta - H\sqrt{R_t} w = 0, \qquad (3.4.1)$$

$$LC - H'\sqrt{R_s} w = 0.$$

Eliminate b from $(3.4.1)_{1,2}$. Thus

$$L^2 w - QD^2 w - a^2\sqrt{R_t}\theta + a^2\sqrt{R_s}C - \frac{1}{N}Lw = 0. \tag{3.4.2}$$

This equation can be written in the form

$$D^4 w_1 - \left(2a_1^{\,2} + \frac{1}{N_1} + Q_1\right)D^2 w_1 + a_1^{\,4} w_1 + a_1^{\,2}\left(\frac{1}{N_1}w_1 - \sqrt{R_{t1}}\theta_1 + \sqrt{R_{s1}}C_1\right) = 0.$$

Multiply by w_2 and integrate to obtain

$$\int D^2 w_2 D^2 w_1 + \left(2a_1^{\,2} + \frac{1}{N_1} + Q_1\right)\int Dw_2 Dw_1 + a_1^{\,4}\int w_2 w_1 + \frac{a_1^{\,2}}{N_1}\int w_2 w_1$$

$$- a_1^{\,2}\sqrt{R_{t1}}\int w_2\theta_1 + a_1^{\,2}\sqrt{R_{s1}}\int w_2 C_1 = 0. \tag{3.4.3}$$

Similarly

$$\int D^2 w_1 D^2 w_2 + \left(2a_2^{\,2} + \frac{1}{N_2} + Q_2\right)\int Dw_1 Dw_2 + a_2^{\,4}\int w_1 w_2 + \frac{a_2^{\,2}}{N_2}\int w_1 w_2$$

$$- a_2^{\,2}\sqrt{R_{t2}}\int w_1\theta_2 + a_2^{\,2}\sqrt{R_{s2}}\int w_1 C_2 = 0. \tag{3.4.4}$$

From $(3.4.1)_3$

$$D^2\theta_1 - a_1^{\,2}\theta_1 - H\sqrt{R_{t1}}w_1 = 0.$$

Multiply by θ_2 and integrate to obtain

$$\int D\theta_2 D\theta_1 + a_1^{\,2}\int\theta_2\theta_1 = -H\sqrt{R_{t1}}\int\theta_2 w_1 \tag{3.4.5}$$

Similarly

$$\int D\theta_1 D\theta_2 + a_2^{\,2}\int\theta_1\theta_2 = -H\sqrt{R_{t2}}\int\theta_1 w_2\,. \tag{3.4.6}$$

From $(3.4.1)_4$

$$D^2C_1 - a_1^2C_1 - H'\sqrt{R_{s1}}w_1 = 0.$$

Multiply by C_2 and integrate to obtain

$$\int DC_2DC_1 + a_1^2\int C_2C_1 = -H'\sqrt{R_{s1}}\int C_2w_1. \tag{3.4.7}$$

Similarly

$$\int DC_1DC_2 + a_2^2\int C_1C_2 = -H'\sqrt{R_{s2}}\int C_1w_2. \tag{3.4.8}$$

By using equations (3.4.6) and (3.4.8), we can eliminate $\int w_2\theta_1$ and $\int w_2C_1$ from equation (3.4.3). Thus

$$\int D^2w_2D^2w_1 + \left(2a_1^2 + \frac{1}{N_1} + Q_1\right)\int Dw_2Dw_1 + a_1^4\int w_2w_1 + \frac{a_1^2}{N_1}\int w_2w_1$$

$$+\frac{a_1^2\sqrt{R_{t1}}}{H\sqrt{R_{t2}}}\left(\int D\theta_2D\theta_1 + a_2^2\int\theta_2\theta_1\right) - \frac{a_1^2\sqrt{R_{s1}}}{H'\sqrt{R_{s2}}}\left(\int DC_2DC_1 + a_2^2\int C_2C_1\right) = 0. \tag{3.4.9}$$

Similarly using equations (3.4.5) and (3.4.7), we can eliminate $\int w_1\theta_2$ and $\int w_1C_2$ from equation (3.4.4). Thus

$$\int D^2w_1D^2w_2 + \left(2a_2^2 + \frac{1}{N_2} + Q_2\right)\int Dw_1Dw_2 + a_2^4\int w_1w_2 + \frac{a_2^2}{N_2}\int w_1w_2$$

$$+\frac{a_2^2\sqrt{R_{t2}}}{H\sqrt{R_{t1}}}\left(\int D\theta_2D\theta_1 + a_1^2\int\theta_2\theta_1\right) - \frac{a_2^2\sqrt{R_{s2}}}{H'\sqrt{R_{s1}}}\left(\int DC_2DC_1 + a_1^2\int C_2C_1\right) = 0. \tag{3.4.10}$$

Subtract equation (3.4.10) from equation (3.4.9) to obtain

$$\left(2a_1^2 - 2a_2^2 + \frac{1}{N_1} - \frac{1}{N_2} + Q_1 - Q_2\right)\int Dw_2 Dw_1 + \left(a_1^4 - a_2^4 + \frac{a_1^2}{N_1} - \frac{a_2^2}{N_2}\right)\int w_2 w_1$$

$$-\frac{1}{H'}\left(\frac{a_1^2 R_{s1} - a_2^2 R_{s2}}{\sqrt{R_{s1}R_{s2}}}\right)\int DC_2 DC_1 - \frac{a_1^2 a_2^2}{H'}\left(\frac{R_{s1} - R_{s2}}{\sqrt{R_{s1}R_{s2}}}\right)\int C_2 C_1$$

$$= -\frac{1}{H}\left(\frac{a_1^2 R_{t1} - a_2^2 R_{t2}}{\sqrt{R_{t1}R_{t2}}}\right)\int D\theta_2 D\theta_1 - \frac{a_1^2 a_2^2}{H}\left(\frac{R_{t1} - R_{t2}}{\sqrt{R_{t1}R_{t2}}}\right)\int \theta_2 \theta_1 \tag{3.4.11}$$

Suppose that $w_1 \to w_2$, $\theta_1 \to \theta_2$, $a_1 \to a_2$, $N_1 \to N_2$, $Q_1 \to Q_2$, $C_1 \to C_2$, $R_{t1} \to R_{t2}$ and $R_{s1} \to R_{s2}$, and divide equation (3.4.11) by $(h_1 - h_2)$ then take the limit as $h_1 \to h_2$. Thus (3.4.11) becomes

$$4a\frac{da}{dh}\left(\int D^2 w + a^2 \int w^2 + \frac{1}{2N}\int w^2 - \frac{1}{2H'}\int D^2 C + \frac{1}{2H}\int D^2\theta\right)$$

$$-\frac{1}{N^2}\frac{dN}{dh}\left(\int D^2 w + a^2 \int w^2\right) + \frac{dQ}{dh}\int D^2 w - \frac{a^2}{H'R_s}\frac{dR_s}{dh}\left(\int D^2 C + a^2 \int C^2\right)$$

$$= -\frac{a^2}{HR_t}\frac{dR_t}{dh}\left(\int D^2\theta + a^2 \int \theta^2\right). \tag{3.4.12}$$

Since

$$R_t = R_t(a, R_s, Q, N)$$

then

$$\frac{dR_t}{dh}=\frac{\partial R_t}{\partial a}\frac{da}{dh}+\frac{\partial R_t}{\partial R_s}\frac{dR_s}{dh}+\frac{\partial R_t}{\partial Q}\frac{dQ}{dh}+\frac{\partial R_t}{\partial N}\frac{dN}{dh}.$$

Thus equation (3.4.12) becomes

$$4a\frac{da}{dh}\left(\int D^2w+a^2\int w^2+\frac{1}{2N}\int w^2-\frac{1}{2H'}\int D^2C+\frac{1}{2H}\int D^2\theta\right)$$

$$-\frac{1}{N^2}\frac{dN}{dh}\left(\int D^2w+a^2\int w^2\right)+\frac{dQ}{dh}\int D^2w-\frac{a^2}{H'R_s}\frac{dR_s}{dh}\left(\int D^2C+a^2\int C^2\right)$$

$$=-\frac{a^2}{HR_t}\left(\frac{\partial R_t}{\partial a}\frac{da}{dh}+\frac{\partial R_t}{\partial R_s}\frac{dR_s}{dh}+\frac{\partial R_t}{\partial Q}\frac{dQ}{dh}+\frac{\partial R_t}{\partial N}\frac{dN}{dh}\right)\left(\int D^2\theta+a^2\int \theta^2\right). \qquad (3.4.13)$$

From this equation, we can show that

$$\frac{\partial R_t}{\partial a}=-\frac{4aHR_t}{\tau}\left(\int D^2w+a^2\int w^2+\frac{1}{2N}\int w^2-\frac{1}{2H'}\int D^2C+\frac{1}{2H}\int D^2\theta\right),$$

$$\frac{\partial R_t}{\partial R_s}=\frac{a^2HR_t}{H'\tau R_s}\left(\int D^2C+a^2\int C^2\right),$$

$$\frac{\partial R_t}{\partial Q}=-\frac{HR_t}{\tau}\int D^2w, \qquad (3.4.14)$$

$$\frac{\partial R_t}{\partial N}=\frac{HR_t}{N^2\tau}\left(\int D^2w+a^2\int w^2\right).$$

where

$$\tau = a^2\left(\int D^2\theta + a^2 \int \theta^2\right).$$

So we have four cases to consider :

Case (1): when the fluid is heated from a bove and the solute concentration decreases upwards.

Here we put $H = 1, \quad H' = -1$ in equations (3.4.14), thus

$$\frac{\partial R_t}{\partial a} = -\frac{4aR_t}{\tau}\left(\int D^2 w + a^2 \int w^2 + \frac{1}{2N}\int w^2 + \frac{1}{2}\int D^2 C + \frac{1}{2}\int D^2\theta\right),$$

$$\frac{\partial R_t}{\partial R_s} = -\frac{a^2 R_t}{\tau\, R_s}\left(\int D^2 C + a^2 \int C^2\right),$$

$$\frac{\partial R_t}{\partial Q} = -\frac{R_t}{\tau}\int D^2 w, \tag{3.4.15}$$

$$\frac{\partial R_t}{\partial N} = \frac{R_t}{\tau\, N^2}\left(\int D^2 w + a^2 \int w^2\right).$$

From equations (3.4.15), it is clear that the thermal Rayleigh number R_t is a decreasing function in a, R_s and Q but an increasing function in N.

Case (2): when the fluid is heated from a bove and the solute concentration increases upwards.

Here $H = 1, \quad H' = 1$ and we deduce from (3.4.14) that

$$\frac{\partial R_t}{\partial a}=-\frac{4aR_t}{\tau}\left(\int D^2w+a^2\int w^2+\frac{1}{2N}\int w^2-\frac{1}{2}\int D^2C+\frac{1}{2}\int D^2\theta\right),$$

$$\frac{\partial R_t}{\partial R_s}=\frac{a^2R_t}{\tau R_s}\left(\int D^2C+a^2\int C^2\right),$$

$$\frac{\partial R_t}{\partial Q}=-\frac{R_t}{\tau}\int D^2w, \qquad (3.4.16)$$

$$\frac{\partial R_t}{\partial N}=\frac{R_t}{\tau N^2}\left(\int D^2w+a^2\int w^2\right)$$

From equation $(3.4.16)_1$, we find that the minimum Rayleigh number over a occurs when

$$\int D^2w+\left(a^2+\frac{1}{2N}\right)\int w^2+\frac{1}{2}\int D^2\theta=\frac{1}{2}\int D^2C.$$

Also from equations $(3.4.16)_{2,3,4}$, it is clear that R_t is an increasing function in R_s and N but a decreasing function in Q .

Case (3): when the fluid is heated from below and the solute concentration increases upwards.

Here $H=-1,\quad H'=1$ and we find from equations (3.4.14) that

$$\frac{\partial R_t}{\partial a}=\frac{4aR_t}{\tau}\left(\int D^2w+a^2\int w^2+\frac{1}{2N}\int w^2-\frac{1}{2}\int D^2C-\frac{1}{2}\int D^2\theta\right),$$

$$\frac{\partial R_t}{\partial R_s}=-\frac{a^2R_t}{\tau\, R_s}\left(\int D^2C+a^2\int C^2\right),$$

$$\frac{\partial R_t}{\partial Q}=\frac{R_t}{\tau}\int D^2w, \tag{3.4.17}$$

$$\frac{\partial R_t}{\partial N}=-\frac{R_t}{\tau\, N^2}\left(\int D^2w+a^2\int w^2\right).$$

From equation $(3.4.17)_1$, we find that the minimum Rayleigh number over a occurs when

$$\int D^2w+\left(a^2+\frac{1}{2N}\right)\int w^2=\frac{1}{2}\left(\int D^2C+\int D^2\theta\right).$$

Also from equations $(3.4.17)_{2,3,4}$, it is clear that R_t is a decreasing function in R_s and N but an increasing function in Q.

Case (4): when the fluid is heated from below and the solute concentration decreases upwards.

Here $H=-1$, $H'=-1$ and equations (3.4.14), become

$$\frac{\partial R_t}{\partial a}=\frac{4aR_t}{\tau}\left(\int D^2w+a^2\int w^2+\frac{1}{2N}\int w^2+\frac{1}{2}\int D^2C-\frac{1}{2}\int D^2\theta\right),$$

$$\frac{\partial R_t}{\partial R_s}=\frac{a^2R_t}{\tau\, R_s}\left(\int D^2C+a^2\int C^2\right),$$

$$\frac{\partial R_t}{\partial Q}=\frac{R_t}{\tau}\int D^2w, \tag{3.4.18}$$

$$\frac{\partial R_t}{\partial N}=-\frac{R_t}{\tau\, N^2}\left(\int D^2w+a^2\int w^2\right)$$

From equation $(3.4.18)_1$, we find that the minimum Rayleigh number occurs when

$$\int D^2w+\left(a^2+\frac{1}{2N}\right)\int w^2+\frac{1}{2}\int D^2C=\frac{1}{2}\int D^2\theta.$$

Also from equations $(3.4.18)_{2,3,4}$, it is clear that R_t is a decreasing function in N but an increasing function in Q and R_s.

3.5 The free boundary problem

In the following analysis we shall consider both boundaries to be free but later on we shall present results for the corresponding rigid boundary value problem. For the free boundary value problem

$$w=D^2w=0 \qquad \text{on } x_3=0\,,\,1,$$

and if we suppose that $w = A\sin(q\pi z)$, is a suitable solution where A is a constant and q is an integer, then

$$Lw = -\lambda w, \text{ where } \lambda = q^2 \pi^2 + a^2 \tag{3.5.1}$$

and (3.3.6) becomes

$$\sigma^4 P_m P_r' P_r + \sigma^3 \left\{ (P_r' P_r + P_m P_r' + P_r P_m + P_m P_r P_r')\lambda + \frac{1}{N} P_m P_r' P_r \right\}$$
$$+ \sigma^2 \left\{ (P_r + P_m + P_r' + P_r' P_r + P_m P_r' + P_m P_r)\lambda^2 + Qq^2\pi^2 P_r P_r' + \frac{1}{N}(P_r' P_r + P_m P_r' + P_r P_m)\lambda \right.$$
$$\left. + a^2 P_m (HR_t P_r' - H'R_s P_r)\lambda^{-1} \right\} + \sigma \left\{ (1 + P_r' + P_r + P_m)\lambda^3 + Qq^2\pi^2 (P_r + P_r')\lambda \right.$$
$$\left. + \frac{1}{N}(P_r + P_m + P_r')\lambda^2 + a^2 (HR_t (P_r' + P_m) - H'R_s (P_r + P_m)) \right\} + \lambda^4 + Qq^2\pi^2 \lambda^2$$
$$+ \frac{1}{N}\lambda^3 + a^2 (HR_t - H'R_s)\lambda = 0. \tag{3.5.2}$$

which is a fourth order polynomial in σ. Since the coefficients of this polynomial are real then its solutions are either all real, two real solutions and two complex conjugate pair solutions or four complex conjugate pair solutions. In the former case if any real solution is positive then stationary instability happens. In the second case if any of the real solutions is positive then stationary instability happens, however if the real part of the complex conjugate pair solutions is positive then overstability happens. In the third case overstability happens if any real part of the complex conjugate pair solutions is positive. The solutions of (3.5.2) are functions of P_r, P_r', P_m, N, Q, R_t, and R_s and we have to examine

how the nature of these solutions depends on these variables by considering the following cases.

Case (1): when the fluid is heated from above and the solute concentration decreases upwards.

Here $H=1,\ H'=-1$. So we need to discuss the roots of the polynomial equation $f(\sigma)=0$ where

$$f(\sigma)=P_mP_rP_r'\sigma^4+\left\{(P_rP_r'+P_mP_r'+P_mP_r+P_mP_rP_r')\lambda+\frac{1}{N}P_mP_rP_r'\right\}\sigma^3$$
$$+\left\{(P_r+P_m+P_r'+P_r'P_r+P_mP_r'+P_mP_r)\lambda^2+Qq^2\pi^2P_rP_r'+\frac{1}{N}(P_rP_r'+P_mP_r'+P_mP_r)\lambda\right.$$
$$\left.+a^2P_m(R_tP_r'+R_sP_r)\lambda^{-1}\right\}\sigma^2+\left\{(1+P_r'+P_r+P_m)\lambda^3+Qq^2\pi^2(P_r+P_r')\lambda\right.$$
$$\left.+\frac{1}{N}(P_r+P_m+P_r')\lambda^2+a^2(R_t(P_r'+P_m)+R_s(P_r+P_m))\right\}\sigma+$$
$$\lambda^4+\frac{1}{N}\lambda^3+Qq^2\pi^2\lambda^2+a^2(R_t+R_s)\lambda. \tag{3.5.3}$$

Clearly all the coefficients of $f(\sigma)$ are positive and real. Thus $f(\sigma)=0$ has either four negative real solutions, two negative real solutions and two complex conjugate pair solutions or four complex conjugate pair solutions. In the first case no instability happens. In the second case we have overstability case since the real part of the complex conjugate per solutions is positive. To prove that let Σ be the sum of the roots of $f(\sigma)=0$ then

$$\Sigma = -\frac{1}{P_m P_r P_r'}\left[\lambda P_r (P_r' + P_m) + \lambda P_m P_r'(1+P_r) + \frac{1}{N} P_m P_r P_r'\right]$$, and we can show that

$$
\begin{aligned}
f(\Sigma) = \lambda^{4}\Bigg\{ & P_r\left[2+\frac{1}{P_m^{2}}+\frac{1}{P_r'^{2}}\right] + (1+P_r)\left[\frac{1}{P_m}\left(2+\frac{P_r'}{P_m}\right)+\frac{1}{P_r'}\left(4+\frac{P_m}{P_r'}\right)\right] \\
& + 2\left[\frac{1}{P_m}\left(P_r\left(1+\frac{1}{P_r'}\right)+1\right)+\left(\frac{P_r'}{P_m}+\frac{P_m}{P_r'}\right)\left(\frac{1}{P_r}+P_r\right)+\left(P_r'+P_m\right)\left(1+\frac{2}{P_r}\right)\right] \\
& + 4\left[2+\frac{1}{P_r}+\frac{P_r'}{P_m}+\frac{P_m}{P_r'}\right]+\frac{1}{P_r}\left[\frac{1}{P_r}\left(P_m+P_r'(1+P_m)\right)+2P_mP_r'+1\right]\Bigg\} \\
+ \frac{\lambda^{3}}{N}\Bigg\{ & P_r\left[\frac{P_r'}{P_m^{2}}+\frac{P_m}{P_r'^{2}}+2\left(\frac{2}{P_m}+\frac{1}{P_r'}\right)+3\right]+\left(1+P_r\right)\left[2\left(\frac{P_m}{P_r'}+2\left(P_r'\left(1+\frac{1}{P_m}\right)\right)\right)+3\right] \\
& + P_r'\left[P_r+\frac{1}{P_r}\left(2+P_m\left(2+\frac{1}{P_r}\right)\right)+4\right]+P_m\left[2\left(\frac{1}{P_r}+4\right)+3\left(P_r'+P_r\right)\right]+2\left[\frac{1}{P_r'}\left(P_r+P_m+P_rP_m\right)\right. \\
& \left. + \frac{1}{P_r}\left(P_r'+P_m+P_r'P_m\right)+3\right]\Bigg\} \\
+ \lambda^{2}\Bigg\{ & Qq^{2}\pi^{2}\left[\frac{P_r'P_r}{P_m}\left(2+\frac{1}{P_m}\right)+\frac{1}{P_m}\left(P_r+P_r'\right)+P_r'\left(1+P_r\right)+P_r+1\right] \\
& + \frac{1}{N^{2}}\left[2\left(P_r\left(\frac{P_r'}{P_m}+\frac{P_m}{P_r'}\right)+\frac{P_mP_r'}{P_r}\right)+4\left(P_r'+P_r+P_m\right)+3\left(P_r\left(P_r'+P_m\right)+P_mP_r'\right)\right]\Bigg\} \\
+ a^{2}\lambda\Bigg\{ & \frac{P_r}{P_r'}\left(1+\frac{P_m}{P_r'}\right)+R_t\left[\left(P_r'+P_m\right)\left(1+\frac{1}{P_r}\right)+P_mP_r'\left(\frac{1}{P_r}\left(2+\frac{1}{P_r}\right)+1\right)+1\right] \\
& + R_s\left[P_r+P_m\left(\frac{1}{P_r'}\left(1+2P_r\right)+1+P_r\right)+1\right] \\
& + \frac{1}{a^{2}N}\left[Qq^{2}\pi^{2}\left(P_r'\left(1+P_r+2P_r\left(1+\frac{1}{P_m}\right)\right)\right)+\frac{1}{N^{2}}\left(P_r'\left(P_r+P_m\right)+P_mP_r\right)\right]\Bigg\} \\
+ \frac{a^{2}}{N}\Bigg\{ & 2P_m\left(\frac{P_r'}{P_r}\left(1+P_r\right)R_t+\frac{P_r}{P_r'}\left(1+P_r'\right)R_s\right)+R_t\left(P_r'+P_m\right)+R_s\left(P_r+P_m\right)\Bigg\} \\
+ \frac{1}{N^{2}}\Big\{ & Qq^{2}\pi^{2}P_r\,P_r'+a^{2}P_m\left(R_tP_r'+R_sP_r\right)\lambda^{-1}\Big\}
\end{aligned}
$$

from which it is obvious that $f(\Sigma)>0$. Also

$$f'(\sigma)=4P_mP_rP_r'\sigma^3+3\sigma^2\left[(P_r'P_r+P_mP_r'+P_mP_r+P_mP_rP_r')\lambda+\frac{1}{N}P_mP_rP_r'\right]$$
$$+2\sigma\left[(P_r+P_m+P_r'+P_r'P_r+P_mP_r'+P_mP_r)\lambda^2+\frac{1}{N}(P_r'P_r+P_mP_r'+P_mP_r)\lambda\right.$$
$$\left.+Qq^2\pi^2P_rP_r'+a^2P_m(R_tP_r'+R_sP_r)\frac{1}{\lambda}\right]$$
$$+\left[(1+P_r'+P_r+P_m)\lambda^3+\frac{1}{N}(P_m+P_r+P_r')\lambda^2+Qq^2\pi^2(P_r+P_r')\lambda\right.$$
$$\left.+a^2(R_t(P_r'+P_m)+R_s(P_r+P_m))\right]>0.$$

i.e. $f(\sigma)$ is an increasing function. Now suppose that σ_1, σ_2 are the real roots and σ_3 is the real part of the two complex conjugate pair roots of $f(\sigma)$. Then

$$\Sigma=\sigma_1+\sigma_2+2\sigma_3$$

since $f(\sigma)$ is an increasing function then

$$\Sigma-\sigma_1 \qquad >0$$

and so

$$\sigma_2+2\sigma_3>0 \qquad \Rightarrow \qquad \sigma_3>0.$$

Thus if $f(\sigma)$ has two real roots and two complex conjugate pair roots then the real part of the complex conjugate roots is positive hence we have overstability. In the third case we suppose that the real parts of the complex conjugate pair roots are σ_1 and σ_2 then

$$\Sigma = 2\sigma_1 + 2\sigma_2 \quad \text{and since} \quad \Sigma < 0 \quad \Rightarrow \sigma_1 + \sigma_2 < 0.$$

If $\sigma_1 < 0$ and $\sigma_2 < 0$ then we have stability.

If $\sigma_1 > 0$ or $\sigma_2 > 0$ then we may have overstability.

Generally, when the fluid is heated from above and solute concentration decreases upwards, no stationary instability happens, however overstability is possible under some conditions. To obtain the critical thermal Rayleigh number for the onest of stationary convection case we set $\sigma = 0$ in (3.5.3).Thus

$$R_t = -\left[\frac{\lambda}{a^2}\left(\lambda^2 + \frac{1}{N}\lambda + Q\pi^2\right) + R_s\right], \qquad q^2 = 1 .$$

Since R_t is negative then no stationary convection happened.

To study the overstability case, we suppose that

$$R_{t1} = \frac{R_t}{\pi^4} \;,\; R_{s1} = \frac{R_s}{\pi^4} \;,\; i\,\sigma_1 = \frac{\sigma}{\pi^2} \;,\; a_1 = \frac{a^2}{\pi^2} \;,\; Q_1 = \frac{Q}{\pi^2} \;,\; N_1 = N\pi^2 \;,\; q^2 = 1.$$

where σ is complex and $\sigma_1 \neq 0$. Thus equation (3.3.5) can be put in the form

$$R_{t1} = -\frac{(1+a_1)(1+a_1+i\sigma_1)(1+a_1+i\sigma_1 P_r)}{a_1} - \frac{(1+a_1+i\sigma_1 P_r)}{(1+a_1+i\sigma_1 P_r')} R_{s1}$$
$$-\frac{(1+a_1)(1+a_1+i\sigma_1 P_r)}{a_1(1+a_1+i\sigma_1 P_m)} Q_1 - \frac{(1+a_1)(1+a_1+i\sigma_1 P_r)}{a_1}\frac{1}{N_1}. \qquad (3.5.4)$$

By equating, separately the real and imaginary parts of equation (3.5.4), we obtain

$$R_{tI}(\zeta-1)(\zeta^{2}-\sigma_{I}^{2}P_{m}P_{r}')=-\zeta\left[\left(\zeta^{2}-\sigma_{I}^{2}P_{r}\right)\left(\zeta^{2}-\sigma_{I}^{2}P_{r}'P_{m}\right)-\zeta^{2}\sigma_{I}^{2}\left(1+P_{r}\right)\left(P_{m}+P_{r}'\right)\right]$$

$$-\left(\zeta-1\right)\left(\zeta^{2}-\sigma_{I}^{2}P_{r}P_{m}\right)R_{sI}-\zeta\left(\zeta^{2}-\sigma_{I}^{2}P_{r}P_{r}'\right)Q_{I}$$

$$-\zeta^{2}\left[\left(\zeta^{2}-\sigma_{I}^{2}P_{r}'P_{m}\right)-\sigma_{I}^{2}P_{r}(P_{r}'+P_{m})\right]\frac{1}{N_{I}}, \qquad (3.5.5)$$

and

$$R_{tI}(\zeta-1)(P_{m}+P_{r}')=-\zeta\left[\left(\zeta^{2}-\sigma_{I}^{2}P_{r}\right)\left(P_{m}+P_{r}'\right)+(1+P_{r})\left(\zeta^{2}-\sigma_{I}^{2}P_{r}'P_{m}\right)\right]$$

$$-\left(\zeta-1\right)\left(P_{r}+P_{m}\right)R_{sI}-\zeta\left(P_{r}+P_{r}'\right)Q_{I}$$

$$-\left[\zeta^{2}\left(P_{m}+P_{r}'\right)+P_{r}\left(\zeta^{2}-\sigma_{I}^{2}\;P_{r}'P_{m}\right)\right]\frac{1}{N_{I}}, \qquad (3.5.6)$$

where $\zeta=1+a_{1}$. Eliminating R_{t1} between equations (3.5.5) and (3.5.6), we obtain

$$P_{r}'^{2}P_{m}^{2}\left[\zeta\left(1+P_{r}\right)+\frac{P_{r}}{N_{I}}\right]Y^{2}+\left\{\zeta^{2}\left(P_{m}^{2}+P_{r}'^{2}\right)\left[\zeta\left(1+P_{r}\right)+\frac{P_{r}}{N_{I}}\right]+\left(\zeta-1\right)\left(P_{r}-P_{r}'\right)P_{m}^{2}R_{sI}\right.$$

$$\left.+\zeta P_{r}'^{2}(P_{r}-P_{m})Q_{I}\right\}Y+\zeta^{2}(\zeta-1)(P_{r}-P_{r}')R_{sI}+\zeta^{3}(P_{r}-P_{m})Q_{I}$$

$$+\zeta^{4}\frac{P_{r}}{N_{I}}+\zeta^{5}(1+P_{r})=0, \qquad (3.5.7)$$

which is a polynomial of order two in $Y=(\sigma_{I}^{2})$ with constant real coefficients. Equation (3.5.7) can be written in the form

$$F_{2}Y^{2}+F_{1}Y+F_{0}=0. \qquad (3.5.8)$$

where

$$F_2 = P_r'^2 P_m^{\ 2}\left[\zeta\,(1+P_r)+\frac{P_r}{N_1}\right],$$

$$F_1 = \zeta^{\ 2}(P_m^{\ 2}+P_r'^2)\left[\zeta\,(1+P_r)+\frac{P_r}{N_1}\right]+(\zeta-1)(P_r-P_r')P_m^{\ 2}R_{s1}+\zeta\,P_r'^2(P_r-P_m)Q_1,$$

$$F_0 = \zeta^{\ 2}(\zeta-1)(P_r-P_r')R_{s1}+\zeta^{\ 3}(P_r-P_m)Q_1+\zeta^{\ 4}\frac{P_r}{N_1}+\zeta^{\ 5}(1+P_r).$$

Consequently,

$$Y=\frac{-F_1\pm\sqrt{F_1^{\ 2}-4F_2F_0}}{2F_2}.$$

Since σ_1 is real and not equal to zero, then Y is real and greater than zero. Consequently, the necessary condition for the overstability case to occur is that $F_0<0$, and this can be obtained if one of the following cases is satisfied

1) $$\frac{\zeta^{\ 3}(1+P_r)+\zeta^{\ 2}\dfrac{P_r}{N_1}-a_1(P_r-P_r')R_{s1}}{\zeta\,(P_r-P_m)}<Q_1, \qquad P_m>P_r,\ P_r'>P_r.$$

2) $$\frac{\zeta^{\ 3}(1+P_r)+\zeta^{\ 2}\dfrac{P_r}{N_1}+a_1(P_r-P_r')R_{s1}}{\zeta\,(P_r-P_m)}<Q_1, \qquad P_r>P_r',\ P_m>P_r. \qquad (3.5.9)$$

3) $$\frac{\zeta^{\ 2}(1+P_r)+\zeta\,\dfrac{P_r}{N_1}}{(P_r-P_m)}<Q_1, \qquad P_r=P_r',\ P_m>P_r.$$

To study the effect of solute concentration, magnetic field and permeability of porous medium on R_{t1} , we have to examine the nature of $\frac{dR_{t1}}{dR_{s1}}$, $\frac{dR_{t1}}{dQ_1}$, and $\frac{dR_{t1}}{dN_1}$. From (3.5.4) we can show that

$$\frac{dR_{t1}}{dR_{s1}} = -\frac{\zeta^2 + \sigma_1^2 P_r P_r' + i\sigma_1\zeta\left(P_r - P_r'\right)}{\zeta^2 + \sigma_1^2 P_r'^2}. \tag{3.5.10}$$

$$\frac{dR_{t1}}{dQ_1} = -\frac{\zeta\left[\zeta^2 + \sigma_1^2 P_r P_m + i\sigma_1\zeta\left(P_r - P_m\right)\right]}{(\zeta - 1)\left(\zeta^2 + \sigma_1^2 P_m^2\right)}. \tag{3.5.11}$$

$$\frac{dR_{t1}}{dN_1} = \frac{\zeta\left(\zeta + i\sigma_1 P_r\right)}{(\zeta - 1)N_1^2}. \tag{3.5.12}$$

By equating separately the real and imaginary parts of equations (3.5.10), (3.5.11) and (3.5.12), we obtain

$$\frac{dR_{t1}}{dR_{s1}} = -\frac{\zeta^2 + \sigma_1^2 P_r P_r'}{\zeta^2 + \sigma_1^2 P_r'^2}, \qquad P_r = P_r'. \tag{3.5.13}$$

$$\frac{dR_{t1}}{dQ_1} = -\frac{\zeta\left(\zeta^2 + \sigma_1^2 P_r P_m\right)}{(\zeta - 1)\left(\zeta^2 + \sigma_1^2 P_m^2\right)}, \qquad P_r = P_m. \tag{3.5.14}$$

$$\frac{dR_{t1}}{dN_1} = \frac{\zeta^2}{(\zeta - 1)N_1^2}. \tag{3.5.15}$$

Hence

$$\frac{dR_{t1}}{dR_{s1}} = -1, \qquad \frac{dR_{t1}}{dQ_1} = -\frac{\zeta}{(\zeta - 1)}, \qquad \frac{dR_{t1}}{dN_1} = \frac{\zeta^2}{(\zeta - 1)N_1^2}.$$

It is clear that R_{tl} is a decreasing function in R_{sl} and Q_l but an increasing function in N_l. Thus for the overstability case the solute concentration and the magnetic field have a destabilizing effect on the system, but the permeability of porous medium has a stabilizing effect on the system.

Case (2): when the fluid is heated from above and the solute concentration increases upwards.

Here $H = 1$, $H' = 1$. To find the thermal Rayleigh number for the onest of stationary convection, we can deduce from equation (3.5.2) that

$$R_t = -\frac{\lambda}{a^2}\left(\lambda^2 + \frac{\lambda}{N} + Q\pi^2\right) + R_s . \tag{3.5.16}$$

It is clear that the stationary instability happens when $R_s > \frac{\lambda}{a^2}\left(\lambda^2 + \frac{\lambda}{N} + Q\pi^2\right)$.

To study the effect of the solute concentration, magnetic field and permeability of porous medium on R_t, we have to examin the nature of $\frac{dR_t}{dR_s}$, $\frac{dR_t}{dQ}$ and $\frac{dR_t}{dN}$. From equation (3.5.16), we obtain

$$\frac{dR_t}{dR_s} = 1 , \qquad \frac{dR_t}{dQ} = -\frac{\lambda\pi^2}{a^2} , \qquad \frac{dR_t}{dN} = \frac{\lambda^2}{a^2 N^2} . \tag{3.5.17}$$

So it is clear that the solute concentration and the permeability of porous medium have a stabilizing effecte on the system, but the magnetic field has a destabilizing effect on the system. The critical value of $R_t (= R_{crit})$ is attained by minimizing R_t over the wave number a . For the overstability case we can show from (3.3.5) that

$$R_{tl} = -\frac{\zeta(\zeta + i\sigma_l)(\zeta + i\sigma_l P_r)}{(\zeta - 1)} + \frac{(\zeta + i\sigma_l P_r)}{(\zeta + i\sigma_l P_r')} R_{sl} - \frac{\zeta(\zeta + i\sigma_l P_r)}{(\zeta - 1)(\zeta + i\sigma_l P_m)} Q_l$$
$$- \frac{\zeta(\zeta + i\sigma_l P_r)}{(\zeta - 1)} \frac{1}{N_l}. \tag{3.5.18}$$

from which we can conclude that overstability occurs if one of the three prementioned cases in (3.5.9) is satisfied. From equation (3.5.18), we obtain

$$\frac{dR_{tl}}{dR_{sl}} = 1 , \quad \frac{dR_{tl}}{dQ_l} = -\frac{\zeta}{(\zeta - 1)} , \quad \frac{dR_{tl}}{dN_l} = \frac{\zeta^2}{(\zeta - 1) N_l^2} .$$

Thus the solute concentration and the permeability of porous medium have a stabilizing effect on the system but the magnetic field has a destabilizing effect on the system.

Case (3): when the fluid is heated from below and the solute concentration increases upwards.

Here $H = -1$, $H' = 1$. So from equation (3.5.2) the thermal Rayleigh number for the onest of stationary convection has the form

$$R_t = \frac{\lambda}{a^2}\left(\lambda^2 + \frac{\lambda}{N} + Q\pi^2\right) - R_s. \tag{3.5.19}$$

It is clear that the stationary convection is possible provided $R_s < \frac{\lambda}{a^2}\left(\lambda^2 + \frac{\lambda}{N} + Q\pi^2\right)$. To study the effect of the solute concentration, magnetic field and permeability of porous medium on R_t, we need to examin the nature of $\frac{dR_t}{dR_s}$, $\frac{dR_t}{dQ}$ and $\frac{dR_t}{dN}$. From equation (3.5.19) we obtain

$$\frac{dR_t}{dR_s} = -1\ , \quad \frac{dR_t}{dQ} = \frac{\lambda\pi^2}{a^2}\ , \quad \frac{dR_t}{dN} = -\frac{\lambda^2}{a^2N^2}. \tag{3.5.20}$$

It is clear from equation (3.5.20) that the solute concentration and the permeability of porous medium have a destabilizing effect on the system but the magnetic field has a stabilizing effect on the system. Following the same procedures as in case one and two we obtain the thermal Rayleigh number for the overstability case,

$$R_{tl} = \frac{\zeta\,(\zeta + i\sigma_l)(\zeta + i\sigma_l P_r)}{(\zeta - 1)} - \frac{(\zeta + i\sigma_l P_r)}{(\zeta + i\sigma_l P_r')}R_{sl} + \frac{\zeta\,(\zeta + i\sigma_l P_r)}{(\zeta - 1)(\zeta + i\sigma_l P_m)}Q_l$$
$$+ \frac{\zeta\,(\zeta + i\sigma_l P_r)}{(\zeta - 1)}\frac{1}{N_l}\ ,$$

from which we conclude that overstability occurs if one of the three prementioned cases in (3.5.9) is satisfied and we can show that

$$\frac{dR_{tl}}{dR_{sl}} = -1\ , \quad \frac{dR_{tl}}{dQ_l} = \frac{\zeta}{(\zeta - 1)}\ , \quad \frac{dR_{tl}}{dN_l} = -\frac{\zeta^2}{(\zeta - 1)N_l^2}.$$

So it is clear that the solute concentration and the permeability of porous medium have a destabilizing effect on the system but the magnetic field has a stabilizing effect on the system.

Case (4): when the fluid is heated from below and the solute concentration decreases upwards.

Here $H = -1$, $H' = -1$. So from equation (3.5.2), the thermal Rayleigh number for the onest of stationary convection has the form

$$R_t = \frac{\lambda}{a^2}\left(\lambda^2 + \frac{\lambda}{N} + Q\pi^2\right) + R_s. \tag{3.5.21}$$

From equation (3.5.21), we obtain

$$\frac{dR_t}{dR_s} = 1, \qquad \frac{dR_t}{dQ} = \frac{\pi^2\lambda}{a^2}, \qquad \frac{dR_t}{dN} = -\frac{\lambda^2}{a^2 N^2}.$$

It is clear that the solute concentration and the magnetic field have a stabilizing effect on the system but the permeability of porous medium has a destabilizing effect on the system. For the overstability case, we can deduce from equation (3.3.5) that

$$R_{tI} = \frac{\zeta(\zeta + i\sigma_I)(\zeta + i\sigma_I P_r)}{(\zeta - 1)} + \frac{(\zeta + i\sigma_I P_r)}{(\zeta + i\sigma_I P_r')} R_{sI} + \frac{\zeta(\zeta + i\sigma_I P_r)}{(\zeta - 1)(\zeta + i\sigma_I P_m)} Q_I$$
$$+ \frac{\zeta(\zeta + i\sigma_I P_r)}{(\zeta - 1)} \frac{1}{N_I}.$$

from which it can be shown that overstability occurs if one of the three prementioned cases in (3.5.9) is satisfied. Following the same procedures we can show that

$$\frac{dR_{tl}}{dR_{sl}}=1\ , \qquad \frac{dR_{tl}}{dQ_l}=\frac{\zeta}{(\zeta-1)}\ , \qquad \frac{dR_{tl}}{dN_l}=-\frac{\zeta^2}{(\zeta-1)N_l^2}\ .$$

So it is clear that the solute concentration and the magnetic field have a stabilizing effect on the system but the permeability of porous medium has a destabilizing effect on the system .

3.6 Numerical results

The eigenvalue problem (3.3.4) together with the boundary conditions are solved numerically for the case when the fluid layer is heated from below and the solute concentration increases upwards using expansion of Chebyshev polynomials. Numerical results were obtained for free and rigid boundaries for the two cases of the solute conditions (3.2.4). Appropriate values of P_r and P_m are given by Chandrasekhar (1952) and suitable values of permeability of porous medium, N, are given by Yamamoto & Iwamura (1976) and Rudraiah, et al. (1980).

The relation between the thermal Rayleigh number, R_t, and the solute Rayleigh number, R_s, for the stationary convection case when both boundaries are free is displayed in figures (2) and (3) for $Q=1000$, $N=0.1$ and $N=0.01$.

We notice from figures (2) and (3) that there is a converse relation between R_t and R_s for case (1) and case (2). Also it appears from these figures that as N decreases (i.e. the porosity decreases) the fluid becomes more stable and so the critical thermal or solutal Rayleigh number increases. The numerical results are illustrated in tables (1)-(4).

The relation between R_t and R_s for the overstability case when both boundaries are free is displayed in figures (4) and (5) for $N = 0.1$ and $N = 0.01$, when $P_r = P_r' = 1$, $P_m = 4$ and $Q = 5000$. It appears from these figures that there is also a converse relation between the thermal Rayleigh number, R_t, and the solute Rayleigh numb, R_s, as in the stationary case.

We notice that the values of R_t and R_s for case (2) are greater than there values in case (1) for the stationary case. However for the overstability case we have the opposite, i.e. the values of R_t and R_s for case (1) are greater than there values in case (2). The numerical results for the overstability case are illustrated in tables (5)-(8).

The relation between the magnetic parameter, Q, and the thermal Rayleigh number, R_t, for the stationary and overstability cases when both boundaries are free is displayed in figure (6) for $N = 0.01$ and $R_s = 3000$. A similar relation between Q and R_s is displayed in figure (7) when $R_t = 3000$. It appears from these figures that as Q increases R_t and R_s increase for both stationary and

overstability cases which means that the presence of magnetic field makes the fluid more stable. The numerical results corresponding to these figures are listed in tables (9)-(12). These results show that the values of R_t and R_s in case (1) are less than there values in case (2) for the stationary case. However for the overstability case the values of R_t and R_s in case (1) are less than those in case (2) for small values of Q, but for large values of Q the values of R_t and R_s in case (1) are greater than those in case (2). The fortran codes for case (1) and case (2) when the boundaries are free are listed in Appendixes (2) and (3) respectively.

The relation between the thermal Rayleigh number, R_t, and the solute Rayleigh number, R_s, for the stationary convection case when both boundaries are rigid is displayed in figures (8) and (9) for $Q=1000$, $N=0.1$ and $N=0.01$. We notice from figures (8) and (9) that there is a converse relation between R_t and R_s for case (1) and case (2). Also it appears from these figures that as N decreases (i.e. the porosity decreases) the fluid becomes more stable and so the critical thermal or solutal Rayleigh number increases. The numerical results are illustrated in tables (13)-(16). The relation between R_t and R_s for the overstability case when both boundaries are rigid is displayed in figures (10) and (11) for $N=0.1$ and $N=0.01$, when $P_r=P_r'=1$, $P_m=4$ and $Q=5000$. It appears from these figures that there is also a converse relation between the

thermal Rayleigh number, R_t, and the solute Rayleigh numb, R_s, as in the stationary case.

We notice that the values of R_t and R_s for case (2) are greater than there values in case (1) for the stationary case. However for the overstability case we have the opposite, i.e. the values of R_t and R_s for case (1) are greater than there values in case (2). The numerical results for the overstability case are illustrated in tables (17)-(20).

The relation between the magnetic parameter, Q, and the thermal Rayleigh number, R_t, for the stationary and overstability cases when both boundaries are rigid is displayed in figure (12) for $N = 0.01$ and $R_s = 3000$. A similar relation between Q and R_s is displayed in figure (13) when $R_t = 3000$. It appears from these figures that as Q increases R_t and R_s increase for both stationary and overstability cases which means that the presence of magnetic field makes the fluid more stable. The numerical results corresponding to these figures are listed in tables (21)-(24). These results show that the values of R_t and R_s in case (1) are less than there values in case (2) for the stationary case. However for the overstability case the values of R_t and R_s in case (1) are greater than those in case(2). The fortran codes for case (1) and case (2) when the boundaries are rigid are listed in Appendixes (4) and (5) respectively.

Table 1: The relation between R_t and R_s for the stationary case when both boundaries are free for $N = 0.1$ and $Q = 1000$.

	Case 1		Case 2	
R_t	a	R_s	a	R_s
1000	3.3420	11664.3527	4.9030	13355.7799
1500	3.5994	11345.9705	4.9351	12913.2017
2000	3.7986	11010.9372	4.9664	12469.6645
2500	3.9623	10663.6869	4.9968	12025.2070
3000	4.1018	10306.9062	5.0264	11579.8656
3500	4.2236	9942.3783	5.0551	11133.6744
4000	4.3320	9571.3714	5.0831	10686.6656
4500	4.4298	9194.8264	5.1104	10238.8693
5000	4.5189	8813.4672	5.1371	9790.3140
5500	4.6009	8427.8659	5.1630	9341.0266
6000	4.6769	8038.4849	5.1884	8891.0325
6500	4.7477	7645.7045	5.2131	8440.3556
7000	4.8140	7249.8427	5.2372	7989.0187
7500	4.8764	6851.1687	5.2608	7537.0431
8000	4.9354	6449.9130	5.2838	7084.4494
8500	4.9912	6046.2747	5.3064	6631.2567
9000	5.0444	5640.4277	5.3284	6177.4834
9500	5.0950	5232.5248	5.3500	5723.1470
10000	5.1434	4822.7010	5.3711	5268.2640

Table 2: The relation between R_t and R_s for the stationary case when both boundaries are free for $N = 0.01$ and $Q = 1000$.

	Case 1		Case 2	
R_t	a	R_s	a	R_s
1000	1.3020	13303.5300	4.1197	16813.8386
1500	1.8928	13177.4489	4.1494	16396.5936
2000	2.2578	12996.5432	4.1785	15978.0486
2500	2.5267	12779.8383	4.2072	15558.2423
3000	2.7410	12537.3095	4.2353	15137.2117
3500	2.9198	12274.9790	4.2629	14714.9930
4000	3.0734	11996.8206	4.2900	14291.6211
4500	3.2084	11705.6228	4.3167	13867.1296
5000	3.3290	11403.4337	4.3428	13441.5513
5500	3.4380	11091.8109	4.3685	13014.9174
6000	3.5375	10771.9722	4.3937	12587.2583
6500	3.6291	10444.8918	4.4185	12158.6031
7000	3.7140	10111.3632	4.4428	11728.9801
7500	3.7932	9772.0439	4.4667	11298.4162
8000	3.8674	9427.4852	4.4902	10866.9376
8500	3.9372	9078.1555	4.5139	10434.5693
9000	4.0031	8724.4566	4.5359	10001.3355
9500	4.0656	8366.7361	4.5581	9567.2595
10000	4.1250	8005.2975	4.5799	9132.3637

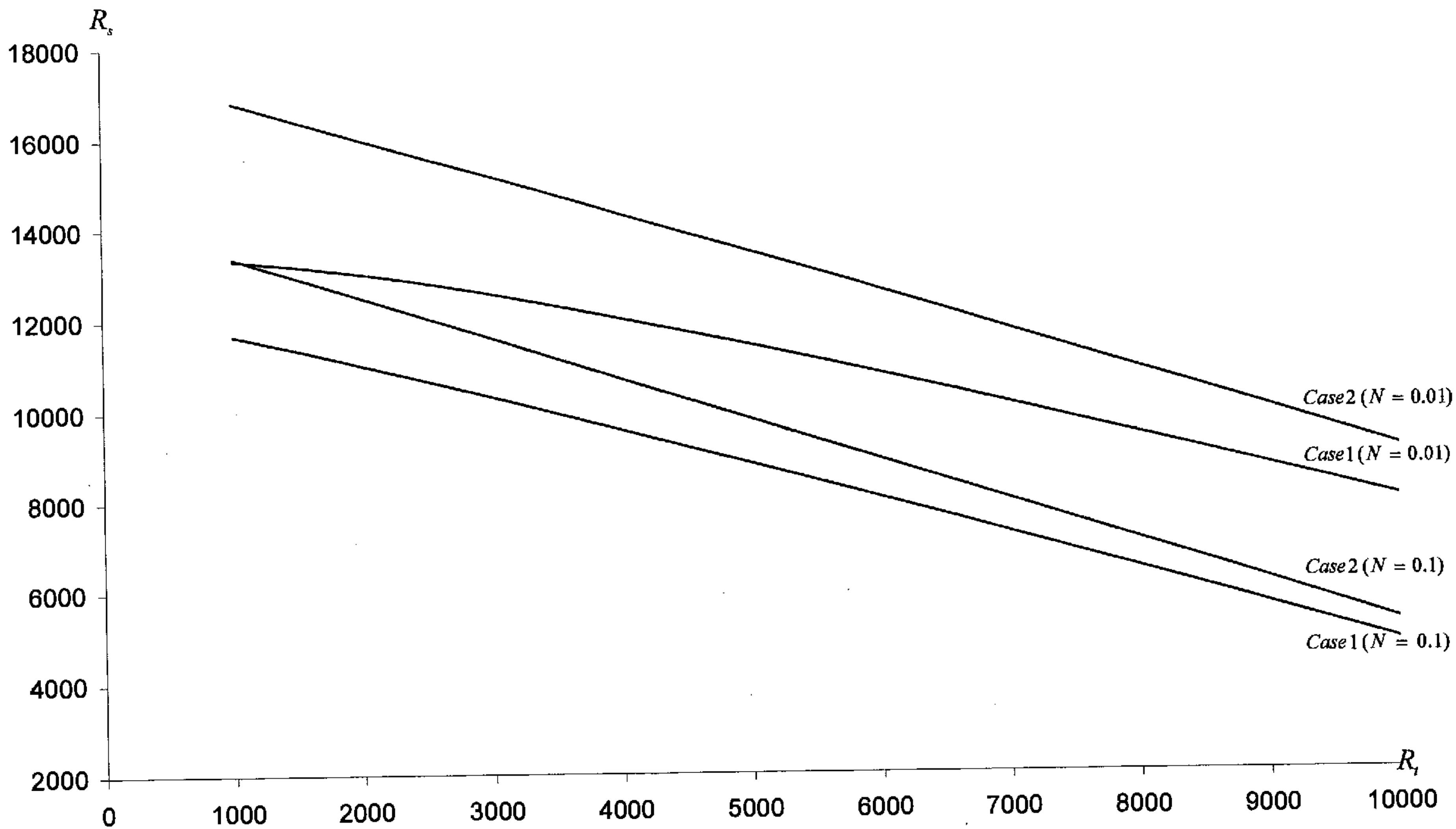


Figure 2: The relation between R_t and R_s for the stationary case when both boundaries are free for $N = 0.1, 0.01$ and $Q=1000$.

Table 3 : The relation between R_s and R_t for the stationary case when both boundaries are free for $N = 0.1$ and $Q = 1000$.

	Case 1		Case 2	
R_s	a	R_t	a	R_t
1000	5.5088	14575.8447	5.5490	14665.5295
1500	5.4678	13984.7824	5.5299	14121.2129
2000	5.4251	13391.7694	5.5103	13576.3472
2500	5.3806	12796.6731	5.4903	13030.9153
3000	5.3340	12199.3446	5.4699	12484.8992
3500	5.2852	11599.6157	5.4490	11938.2799
4000	5.2339	10997.2950	5.4276	11391.0375
4500	5.1799	10392.1630	5.4058	10843.1507
5000	5.1227	9783.9665	5.3834	10294.5973
5500	5.0621	9172.4096	5.3604	9745.3535
6000	4.9974	8557.1440	5.3369	9195.3943
6500	4.9282	7937.7537	5.3128	8644.6929
7000	4.8536	7313.7351	5.2881	8093.2211
7500	4.7726	6684.4684	5.2627	7540.9485
8000	4.6841	6049.1751	5.2366	6987.8432
8500	4.5862	5406.8533	5.2098	6433.8706
9000	4.4764	4756.1742	5.1823	5878.9940
9500	4.3514	4095.3059	5.1539	5323.1741
10000	4.2055	3421.5896	5.1247	4766.3685

Table 4 : The relation between R_s and R_t for the stationary case when both boundaries are free for $N = 0.01$ and $Q = 1000$.

	Case 1		**Case 2**	
R_s	a	R_t	a	R_t
1000	4.8839	19084.9480	4.9235	19207.1690
1500	4.8444	18461.5608	4.9054	18647.6522
2000	4.8034	17835.4062	4.8870	18087.3946
2500	4.7608	17206.3021	4.8681	17526.3749
3000	4.71640	16574.0452	4.8489	16964.5709
3500	4.6702	15938.4077	4.8294	16401.9592
4000	4.6219	15299.1323	4.8094	15838.5155
4500	4.5713	14655.9274	4.7891	15274.2141
5000	4.5181	14008.4594	4.7684	14709.0282
5500	4.4622	13356.3444	4.7473	14142.9297
6000	4.4032	12699.1365	4.7257	13575.8890
6500	4.3405	12036.3130	4.7037	13007.8749
7000	4.2739	11367.2541	4.6813	12438.8547
7500	4.2025	10691.2160	4.6583	11868.7940
8000	4.1258	10007.2922	4.6349	11297.6565
8500	4.0428	9314.3598	4.6110	10725.4038
9000	3.9521	8610.9981	4.5865	10151.9956
9500	3.8522	7895.3654	4.5615	9577.3893
10000	3.7407	7164.9986	4.5359	9001.5398

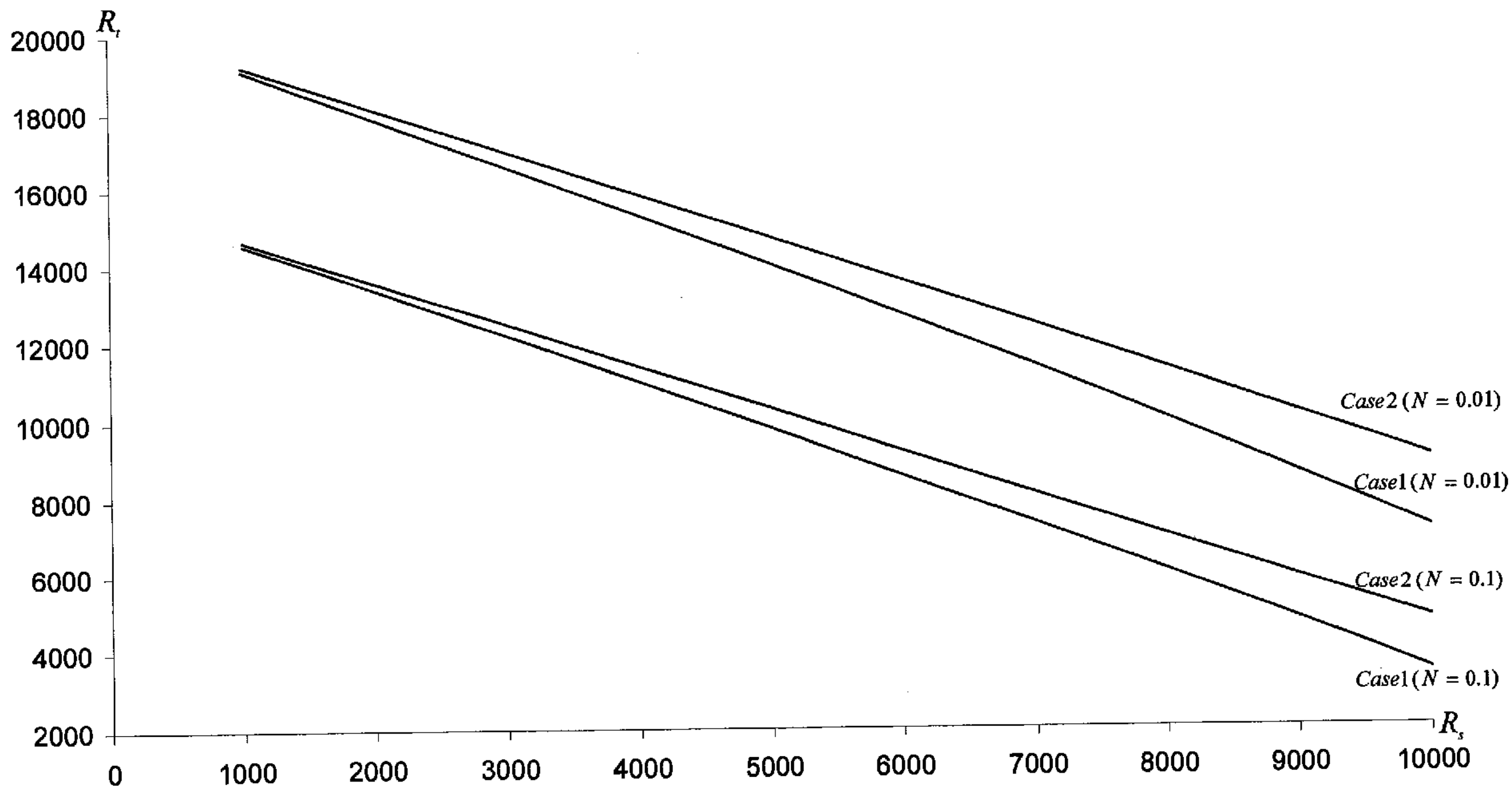


Figure 3: The relation between R_s and R_t for the stationary case when both boundaries are free for $N = 0.1, 0.01$ and $Q = 1000$.

Table 5 : The relation between R_t and R_s for overstability case when both boundaries are free for $N = 0.1$, $Q = 5000$, $P_r = P_r' = 1$ and $P_m = 4$.

	Case 1		Case 2	
R_t	a	R_s	a	R_s
1000	5.4244	14701.2191	5.2697	14268.0491
1500	5.4135	14171.7182	5.2644	13751.7293
2000	5.4026	13641.9830	5.2590	13235.3861
2500	5.3916	13112.0161	5.2537	12719.0197
3000	5.3807	12581.8203	5.2484	12202.6308
3500	5.3697	12051.3983	5.2430	11686.2196
4000	5.3587	11520.7530	5.2378	11169.7867
4500	5.3478	10989.8874	5.2325	10653.3323
5000	5.3368	10458.8045	5.2272	10136.8569
5500	5.3258	9927.5074	5.2220	9620.3610
6000	5.3148	9395.9994	5.2167	9103.8448
6500	5.3039	8864.2836	5.2115	8587.3089
7000	5.2930	8332.3635	5.2063	8070.7535
7500	5.2821	7800.2422	5.2011	7554.1791
8000	5.2712	7267.9233	5.1959	7037.5861
8500	5.2604	6735.4103	5.1908	6520.9748
9000	5.2495	6202.7067	5.1856	6004.3456
9500	5.2387	5669.8159	5.1805	5487.6989
10000	5.2280	5136.7417	5.1754	4971.0351

Table 6: The relation between R_t and R_s for overstability case when both boundaries are free for $N = 0.01$, $Q = 5000$, $P_r = P_r' = 1$ and $P_m = 4$.

	Case 1		Case 2	
R_t	a	R_s	a	R_s
1000	5.1224	22785.3095	5.0348	22605.7571
1500	5.1190	22279.4071	5.0322	22099.3252
2000	5.1156	21773.2312	5.0296	21592.8442
2500	5.1122	21266.7832	5.0270	21086.3148
3000	5.1086	20760.0644	5.0244	20579.7373
3500	5.1050	20253.0762	5.0218	20073.1122
4000	5.1012	19745.8200	5.0192	19566.4399
4500	5.0974	19238.2972	5.0165	19059.7209
5000	5.0936	18730.5091	5.0139	18552.9557
5500	5.0896	18222.4571	5.0112	18046.1447
6000	5.0856	17714.1428	5.0086	17539.2883
6500	5.0815	17205.5674	5.0059	17032.3870
7000	5.0773	16696.7325	5.0033	16525.4413
7500	5.0731	16187.6396	5.0006	16018.4516
8000	5.0688	15678.2901	4.9979	15511.4184
8500	5.0644	15168.6855	4.9952	15004.3421
9000	5.0600	14658.8273	4.9925	14497.2231
9500	5.0555	14148.7172	4.9898	13990.0619
10000	5.0509	13638.3566	4.9871	13482.8589

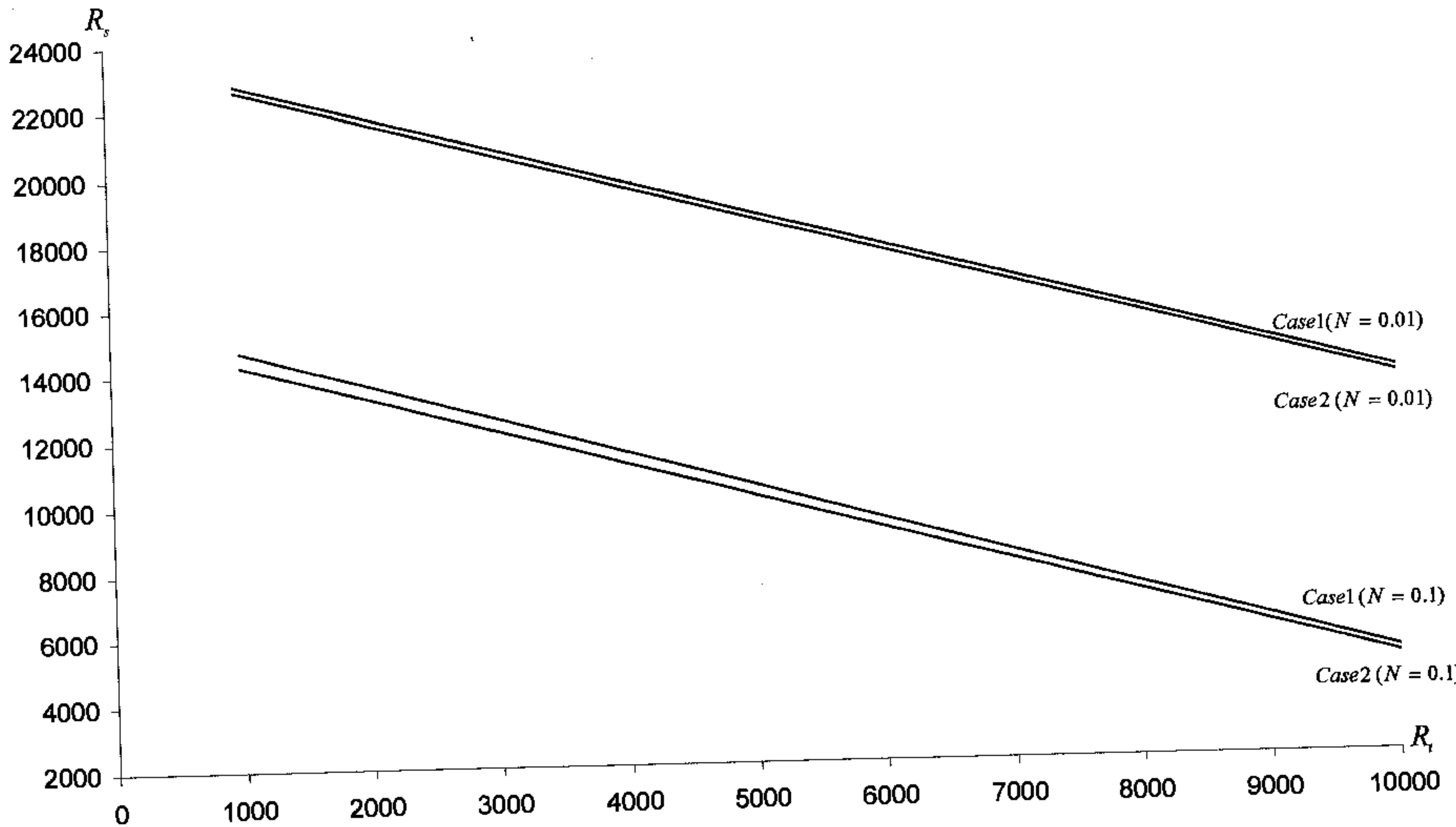


Figure 4 : The relation between R_t and R_s for overstability case when both boundaries are free for $N = 0.1, 0.01$, $Q = 5000$, $P_r = P_r' = 1$ and $P_m = 4$.

Table 9: The relation between Q and R_t for the stationary case when both boundaries are free for $N = 0.01$ and $R_s = 3000$.

	Case 1		Case 2	
Q	a	R_t	a	R_t
300	3.2329	5541.2850	3.7073	6410.0219
450	3.7587	8211.8569	4.0584	8860.5595
600	4.1071	10636.0694	4.3284	11172.5337
750	4.3731	12929.1020	4.5494	13394.9371
900	4.5903	15137.0382	4.7373	15553.3569
1050	4.7751	17283.9014	4.9013	17663.1116
1200	4.9365	19384.1154	5.0472	19734.2231
1350	5.0983	21447.4230	5.1789	21773.6708
1500	5.2099	23497.6342	5.2991	23786.5459
1650	5.3283	25486.4062	5.4098	25776.6979
1800	5.4376	27471.1441	5.5124	27747.1212
1950	5.5390	29436.7147	5.6083	29700.2004
2100	5.6336	31385.4061	5.6982	31637.8709
2250	5.7225	33319.0798	5.7830	33561.7798
2400	5.8063	35239.2745	5.8631	35473.1133
2550	5.8856	37147.2798	5.9393	37373.1528
2700	5.9610	39044.1896	6.0118	39262.8167
2850	6.0328	40930.9411	6.0810	41142.9413
3000	6.1013	42808.3452	6.1473	43014.2550

Table 10 : The relation between Q and R_t for overstability case when both boundaries are free for $N = 0.01$, $R_s = 3000$, $P_r = P_r' = 1$ and $P_m = 4$.

	Case 1		Case 2	
Q	a	R_t	a	R_t
300	3.1249	3860.0550	3.1475	4170.8591
450	3.3114	4706.1984	3.2991	4908.3980
600	3.4487	5450.2800	3.4225	5588.6143
750	3.5600	6133.7739	3.5283	6230.5614
900	3.6557	6776.8677	3.6221	6844.7317
1050	3.7408	7390.8234	3.7071	7437.5457
1200	3.8182	7982.5478	3.7852	8013.2411
1350	3.8897	8556.5865	3.8577	8574.7796
1500	3.9565	9116.0997	3.9256	9124.3255
1650	4.0193	9663.3823	3.9896	9663.5181
1800	4.0788	10200.1608	4.0502	10193.6350
1950	4.1354	10727.7730	4.1078	10715.6966
2100	4.1894	11247.2806	4.1627	11230.5338
2250	4.2410	11759.5440	4.2153	11738.8352
2400	4.2906	12265.2722	4.2658	12241.1798
2550	4.3384	12765.0590	4.3144	12738.0599
2700	4.3844	13259.4070	4.3612	13229.8994
2850	4.4289	13748.7480	4.4063	13717.0668
3000	4.4719	14233.4551	4.4500	14199.8843

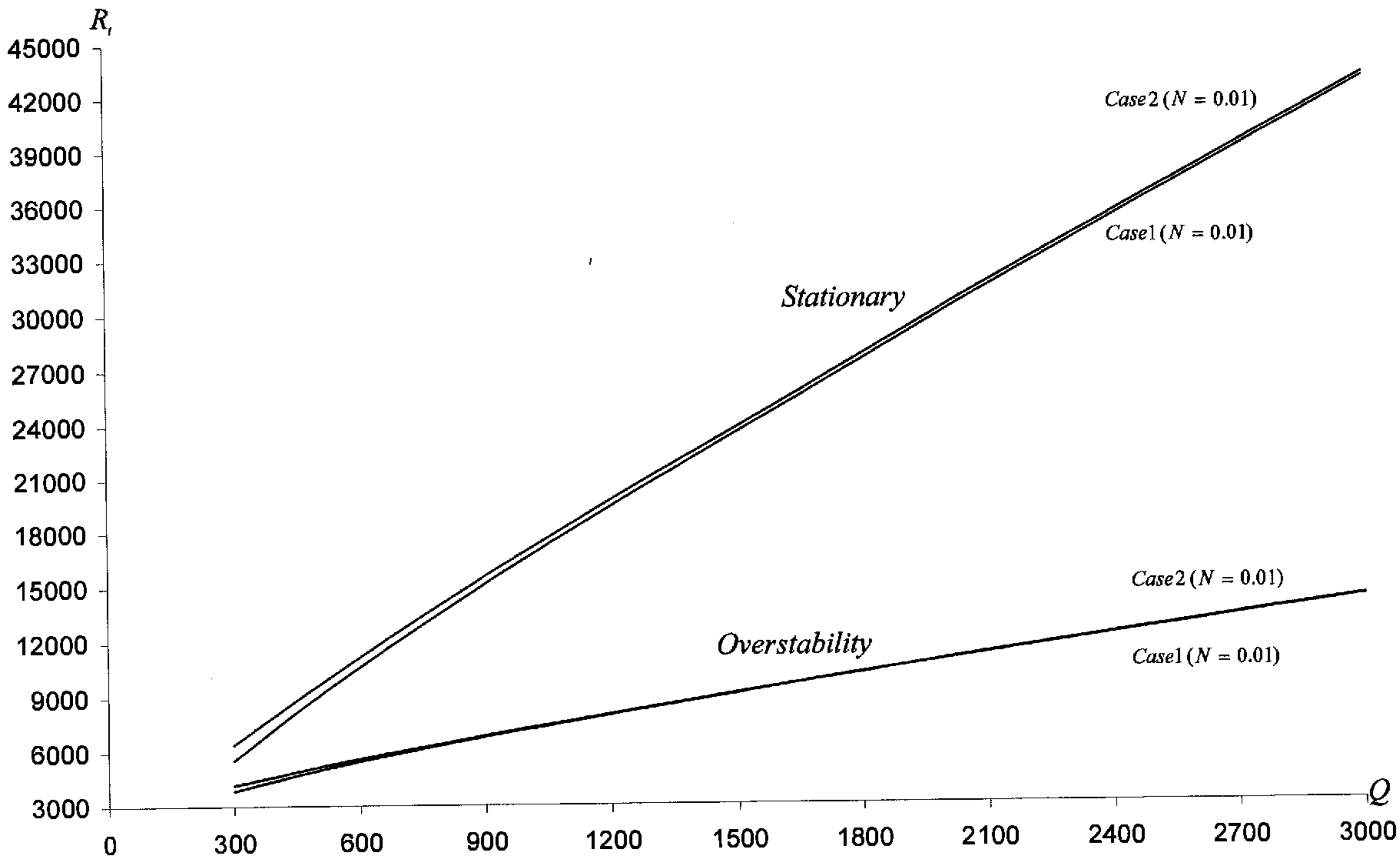


Figure 6: The relation between Q and R_t for the stationary and overstability cases when both boundaries are free for $N = 0.01$ and $R_s = 3000$.

Table 11: The relation between Q and R_s for the stationary case when both boundaries are free for $N = 0.01$ and $R_t = 3000$.

	Case 1		Case 2	
Q	a	R_s	a	R_s
300	2.4452	4360.8359	3.4118	5699.7470
450	2.5089	6117.2540	3.6413	7798.4218
600	2.5724	7871.1770	3.8327	9845.4184
750	2.6358	9622.8042	3.9984	11853.9951
900	2.6990	11372.2275	4.1454	13832.3285
1050	2.7619	13119.4940	4.2781	15785.9350
1200	2.8246	14864.6306	4.3993	17718.7687
1350	2.8870	16607.6544	4.5112	19633.7916
1500	2.9491	18348.5778	4.6152	21533.2976
1650	3.0108	20087.4116	4.7125	23419.1102
1800	3.0721	21824.1664	4.8040	25292.7102
1950	3.1331	23558.8534	4.8903	27155.3212
2100	3.1936	25291.4853	4.9723	29007.9692
2250	3.2535	27022.0761	5.0501	30851.5255
2400	3.3129	28750.6416	5.1245	32686.7378
2550	3.3717	30477.0543	5.1956	34514.2540
2700	3.4300	32201.6190	5.2637	36334.6402
2850	3.4876	33924.3688	5.3291	38148.3946
3000	3.5446	35645.0238	5.3921	39955.9584

Table 12 : The relation between Q and R_s for overstability case when both boundaries are free for $N = 0.01$, $R_t = 3000$, $P_r = P_r' = 1$ and $P_m = 4$.

	Case 1		Case 2	
Q	a	R_s	a	R_s
300	3.0741	3664.1639	3.1205	4040.3086
450	3.2486	4405.1567	3.2759	4750.7562
600	3.3931	5108.1253	3.4085	5427.3573
750	3.5183	5782.8005	3.5250	6077.4653
900	3.6297	6434.9006	3.6293	6705.9389
1050	3.7307	7068.2123	3.7240	7316.2455
1200	3.8232	7685.4453	3.8109	7910.9866
1350	3.9089	8288.646	3.8913	8492.1837
1500	3.9888	8879.4210	3.9662	9061.4503
1650	4.0636	9459.0691	4.0364	9620.1004
1800	4.1342	10028.6638	4.1025	10169.2227
1950	4.2008	10589.1085	4.1650	10709.7313
2100	4.2641	11141.1742	4.2243	11242.4033
2250	4.3244	11685.5270	4.2808	11767.9056
2400	4.3819	12222.7477	4.3347	12286.8157
2550	4.4369	12753.3474	4.3864	12799.6376
2700	4.4896	13277.7787	4.4359	13306.8145
2850	4.5402	13796.4452	4.4836	13808.7381
3000	4.5889	14309.7086	4.5295	14305.7569

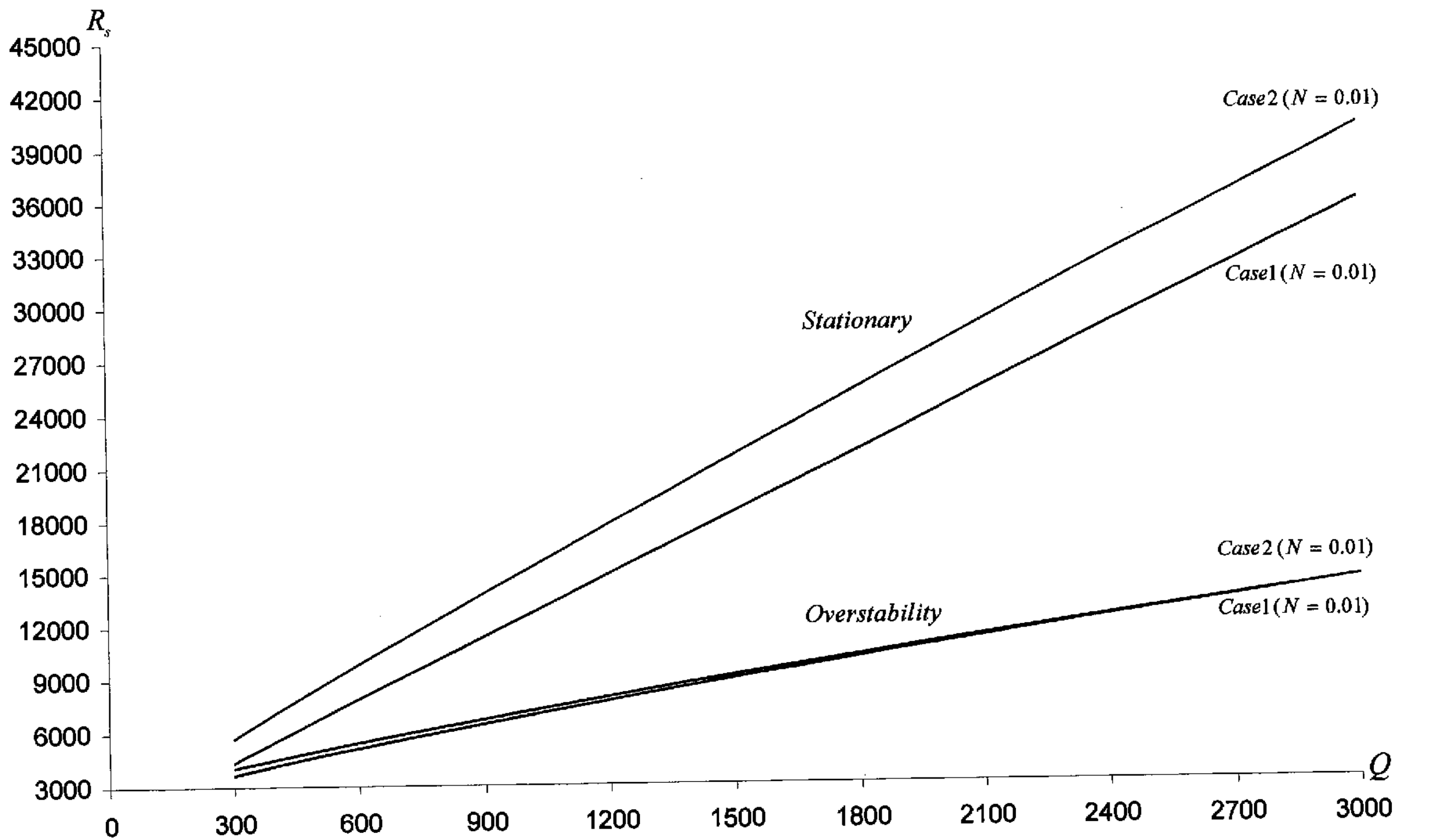


Figure 7: The relation between Q and R_s for the stationary and overstability cases when both boundaries are free for $N = 0.01$ and $R_t = 3000$.

Table 13 : The relation between R_t and R_s for the stationary case when both boundaries are rigid for $N = 0.1$, $Q = 1000$.

	Case 1		**Case 2**	
R_t	a	R_s	a	R_s
1000	3.7100	13824.7269	5.0932	15408.6892
1500	3.9006	13470.3537	5.1190	14954.1690
2000	4.0558	13104.8817	5.1440	14499.0097
2500	4.1875	12730.8221	5.1803	14043.2932
3000	4.3021	12349.2969	5.1925	13586.8636
3500	4.4040	11963.0258	5.2161	13129.9184
4000	4.4958	11571.2952	5.2229	12672.5272
4500	4.5794	11175.2789	5.2611	12214.3795
5000	4.6563	10775.4910	5.2829	11755.8211
5500	4.7276	10372.3456	5.3043	11296.7589
6000	4.7939	9966.1832	5.3252	10837.2083
6500	4.8561	9557.2878	5.3453	10377.1842
7000	4.9146	9145.8998	5.3659	9916.7002
7500	4.9699	8732.2247	5.3855	9455.7701
8000	5.0224	8316.4402	5.4048	8994.4064
8500	5.0722	7898.7011	5.4243	8532.6214
9000	5.1197	7479.1436	5.4423	8070.4263
9500	5.1652	7057.8880	5.4605	7607.8325
10000	5.2087	6635.0413	5.4783	7144.8506

Table 14 : The relation between R_t and R_s for the stationary case when both boundaries are rigid for $N = 0.01$ and Q=1000.

	Case 1		Case 2	
R_t	a	R_s	a	R_s
1000	1.7899	15793.5219	4.3069	19146.4297
1500	2.2327	15611.5204	4.3322	18713.9358
2000	2.5331	15386.3044	4.3570	18280.4936
2500	2.7601	15132.0855	4.3814	17846.1306
3000	2.9494	14856.5618	4.4054	17410.8733
3500	3.1073	14564.4956	4.4290	16974.7471
4000	3.2443	14259.0769	4.4521	16537.7768
4500	3.3655	13942.5749	4.4748	16099.9858
5000	3.4743	13616.6735	4.4972	15661.3971
5500	3.5731	13282.6646	4.5192	15222.0326
6000	3.6636	12941.5655	4.5372	14781.9220
6500	3.7472	12594.1951	4.5620	14341.0598
7000	3.8249	12241.2241	4.5829	13899.4914
7500	3.8975	11883.2106	4.6034	13457.2271
8000	3.9657	11520.6248	4.6234	13014.2851
8500	4.0300	11153.8674	4.6435	12570.6828
9000	4.0908	10783.2835	4.6630	12126.4370
9500	4.1484	10409.1723	4.6822	11681.5640
10000	4.2033	10031.7961	4.7011	11236.0794

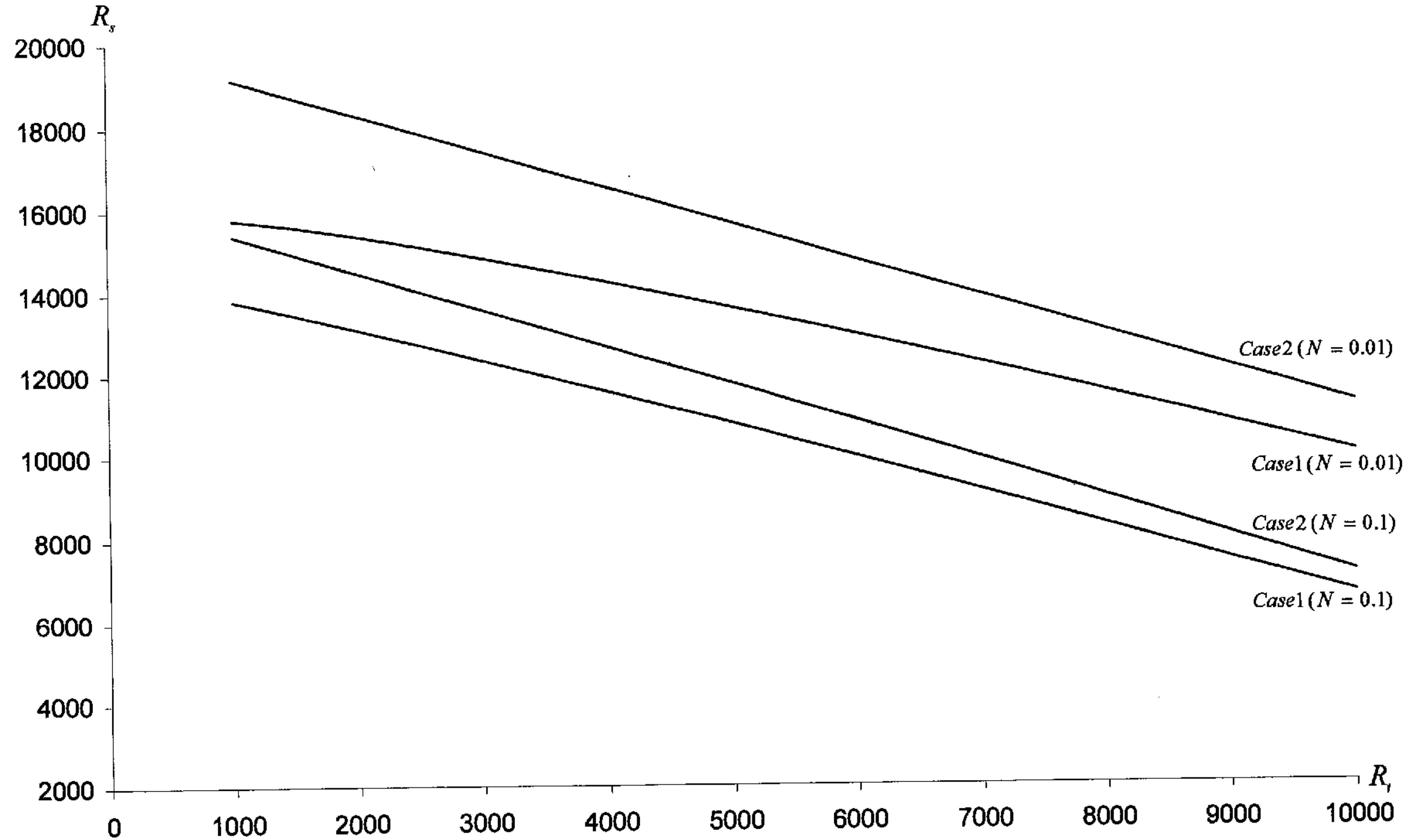


Figure 8 : The relation between R_t and R_s for the stationary case when both boundaries are rigid for $N = 0.1, 0.01$, $Q = 1000$.

Table 15: The relation between R_s and R_t for the stationary case when both boundaries are rigid for $N = 0.1$ and Q=1000.

	Case 1		**Case 2**	
R_s	a	R_t	a	R_t
1000	5.6556	16530.6380	5.6876	16601.4560
1500	5.6225	15958.9734	5.6722	16066.4559
2000	5.5896	15386.0533	5.6566	15531.1084
2500	5.5549	14811.8103	5.6408	14995.4042
3000	5.5190	14236.1691	5.6246	14459.3345
3500	5.4817	13659.0472	5.6083	13922.8893
4000	5.4429	13080.3536	5.5914	13386.0586
4500	5.4026	12499.9868	5.5743	12848.8318
5000	5.3605	11917.8339	5.5569	12311.1977
5500	5.3166	11333.7679	5.5388	11773.1449
6000	5.2705	10747.6457	5.5211	11234.6608
6500	5.2222	10159.3048	5.5015	10695.7337
7000	5.1712	9568.5590	5.4838	10156.3479
7500	5.1174	8975.1936	5.4645	9616.4914
8000	5.0604	8378.9590	5.4450	9076.1486
8500	4.9996	7779.5610	5.4249	8535.3039
9000	4.9345	7176.6499	5.4045	7993.9407
9500	4.8645	6569.8033	5.3836	7452.0414
10000	4.7886	5958.5027	5.3622	6909.5882

Table 16 : The relation between R_s and R_t for the stationary case when both boundaries are rigid for $N=0.01$ and $Q=1000$.

	Case 1		Case 2	
R_s	a	R_t	a	R_t
1000	5.0299	21237.1754	5.0622	21335.6592
1500	4.9978	20637.9839	5.0472	20787.5882
2000	4.9646	20036.9456	5.0320	20239.0266
2500	4.9305	19433.9611	5.0167	19689.9619
3000	4.8952	18828.9219	5.0008	19140.3824
3500	4.8586	18221.7091	4.9850	18590.2750
4000	4.8208	17612.1915	4.9688	18039.6263
4500	4.7816	17000.2242	4.9524	17488.4224
5000	4.7408	16385.6466	4.9357	16936.6487
5500	4.6984	15768.2793	4.9187	16384.2903
6000	4.6542	15147.9214	4.9024	15831.3319
6500	4.6081	14524.3465	4.8840	15277.7550
7000	4.5597	13897.2980	4.8661	14723.5445
7500	4.5089	13266.4829	4.8480	14168.6822
8000	4.4555	12631.5646	4.8294	13613.1488
8500	4.3991	11992.1524	4.8102	13056.9251
9000	4.3394	11347.7894	4.7914	12499.9902
9500	4.2758	10697.9348	4.7719	11942.3231
10000	4.2078	10041.9410	4.7520	11383.9010

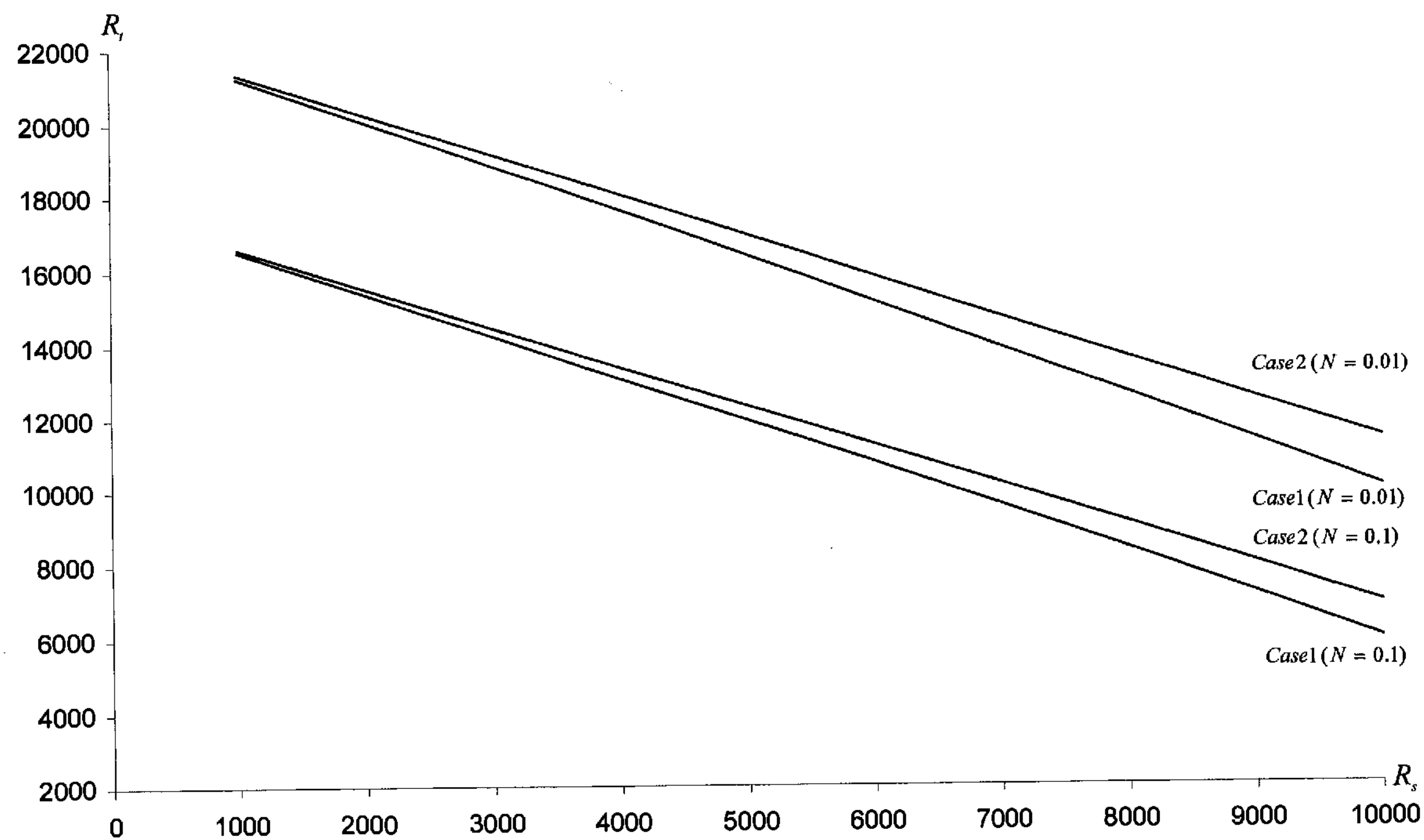


Figure 9: The relation between R_s and R_t for the stationary case when both boundaries are rigid for $N = 0.1, 0.01$ and Q=1000.

Table 17: The relation between R_t and R_s for the overstability case when both boundaries are rigid for $N = 0.1$, Q=5000 , $P_r = P_r' = 1$ and $P_m = 4$.

	Case 1		Case 2	
R_t	a	R_s	a	R_s
1000	7.2048	32218.3074	7.1253	31623.0791
1500	7.2021	31699.1886	7.1242	31113.5638
2000	7.1995	31180.0736	7.1230	30604.0541
2500	7.1969	30660.9628	7.1218	30094.5500
3000	7.1944	30141.8562	7.1206	29585.0514
3500	7.1918	29622.7539	7.1195	29075.5584
4000	7.1891	29103.6560	7.1183	28566.0710
4500	7.1866	28584.5627	7.1171	28056.5892
5000	7.1840	28065.4740	7.1159	27547.1129
5500	7.1815	27546.3900	7.1148	27037.6422
6000	7.1789	27027.3110	7.1136	26528.1771
6500	7.1765	26508.2369	7.1124	26018.7175
7000	7.1738	25989.1679	7.1113	25509.2635
7500	7.1713	25470.1041	7.1101	24999.8150
8000	7.1688	24951.0456	7.1089	24490.3721
8500	7.1662	24431.9926	7.1078	23980.9347
9000	7.1637	23912.945	7.1066	23471.5028
9500	7.1612	23393.9031	7.1055	22962.0765
10000	7.1587	22874.8669	7.1042	22452.6558

Table 18: The relation between R_t and R_s for the overstability case when both boundaries are rigid for $N = 0.01$, Q=5000 , $P_r = P_r' = 1$ and $P_m = 4$.

	Case 1		Case 2	
R_t	a	R_s	a	R_s
1000	6.9357	44783.5305	6.8278	43993.4360
1500	6.9329	44265.2720	6.8265	43483.6836
2000	6.9304	43746.9999	6.8252	42973.9359
2500	6.9279	43228.7141	6.8240	42464.1931
3000	6.9253	42710.4148	6.8227	41954.4552
3500	6.9228	42192.1021	6.8213	41444.7221
4000	6.9203	41673.7762	6.8201	40934.9938
4500	6.9178	41155.4371	6.8188	40425.2704
5000	6.9153	40637.0849	6.8175	39915.5518
5500	6.9128	40118.7199	6.8163	39405.8382
6000	6.9099	39600.3423	6.8150	38896.1293
6500	6.9078	39081.9516	6.8137	38386.4254
7000	6.9052	38563.5486	6.8124	37876.7264
7500	6.9027	38045.1332	6.8112	37367.0322
8000	6.9002	37526.7056	6.8099	36857.3429
8500	6.8977	37008.2658	6.8086	36347.6586
9000	6.8952	36489.8140	6.8073	35837.9791
9500	6.8926	35971.3503	6.8060	35328.3046
10000	6.8901	35452.8749	6.8047	34818.6350

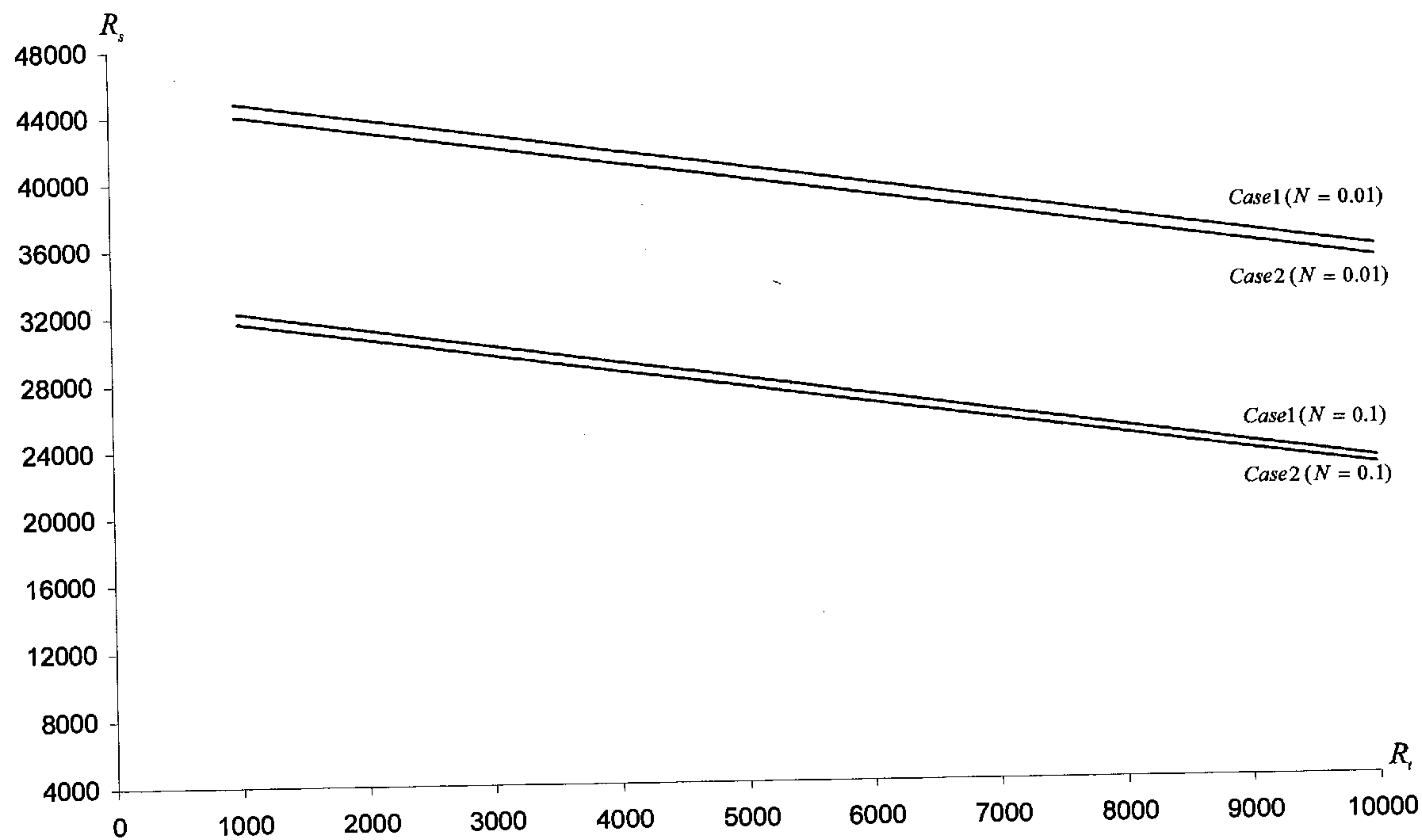


Figure 10 : The relation between R_t and R_s for the overstability case when both boundaries are rigid for $N = 0.1, 0.01$, $Q=5000$, $P_r = P'_r = 1$ and $P_m = 4$.

Table 19: The relation between R_s and R_t for the overstability case when both boundaries are rigid for $N = 0.1$, Q=5000 , $P_r = P_r' = 1$ and $P_m = 4$.

	Case 1		Case 2	
R_s	a	R_t	a	R_t
1000	7.0594	31078.8382	7.0572	31060.8272
1500	7.0616	30596.8732	7.0581	30569.8517
2000	7.0637	30114.9167	7.0595	30078.8811
2500	7.0658	29632.9685	7.0605	29587.9157
3000	7.0680	29151.0287	7.0615	29096.9555
3500	7.0702	28669.0971	7.0626	28606.0004
4000	7.0723	28187.1737	7.0637	28115.0504
4500	7.0745	27705.2584	7.0647	27624.1055
5000	7.0767	27223.3512	7.0658	27133.1657
5500	7.0789	26741.4520	7.0669	26642.2311
6000	7.0810	26259.5607	7.0680	26151.3016
6500	7.0832	25777.6773	7.0689	25660.3773
7000	7.0854	25295.8016	7.0700	25169.4582
7500	7.0877	24813.9337	7.0712	24678.5441
8000	7.0899	24332.0734	7.0723	24187.6352
8500	7.0921	23850.2208	7.0734	23696.7314
9000	7.0943	23368.3756	7.0746	23205.8329
9500	7.0966	22886.5379	7.0756	22714.9394
10000	7.0988	22404.7075	7.0767	22224.0512

Table 20: The relation between R_s and R_t for the overstability case when both boundaries are rigid for $N = 0.01$, $Q = 5000$, $P_r = P_r' = 1$ and $P_m = 4$.

	Case 1		Case 2	
R_s	a	R_t	a	R_t
1000	6.7256	43206.2878	6.7234	43188.0527
1500	6.7279	42724.5558	6.7249	42697.2164
2000	6.7303	42242.8212	6.7256	42206.3810
2500	6.7326	41761.0837	6.7269	41715.5526
3000	6.7349	41279.3433	6.7280	41224.7291
3500	6.7372	40797.5999	6.7292	40733.9103
4000	6.7395	40315.8534	6.7304	40243.0965
4500	6.7419	39834.1037	6.7315	39752.2877
5000	6.7442	39352.3505	6.7327	39261.4837
5500	6.7466	38870.5940	6.7338	38770.6846
6000	6.7489	38388.8338	6.7350	38279.8904
6500	6.7512	37907.0699	6.7361	37789.1011
7000	6.7536	37425.3023	6.7374	37298.3167
7500	6.7560	36943.5307	6.7386	36807.5371
8000	6.7583	36461.7552	6.7398	36316.7625
8500	6.7606	35979.9754	6.7408	35825.9927
9000	6.7630	35498.1915	6.7421	35335.2278
9500	6.7654	35016.4031	6.7435	34844.4678
10000	6.7677	34534.6103	6.7444	34353.7126

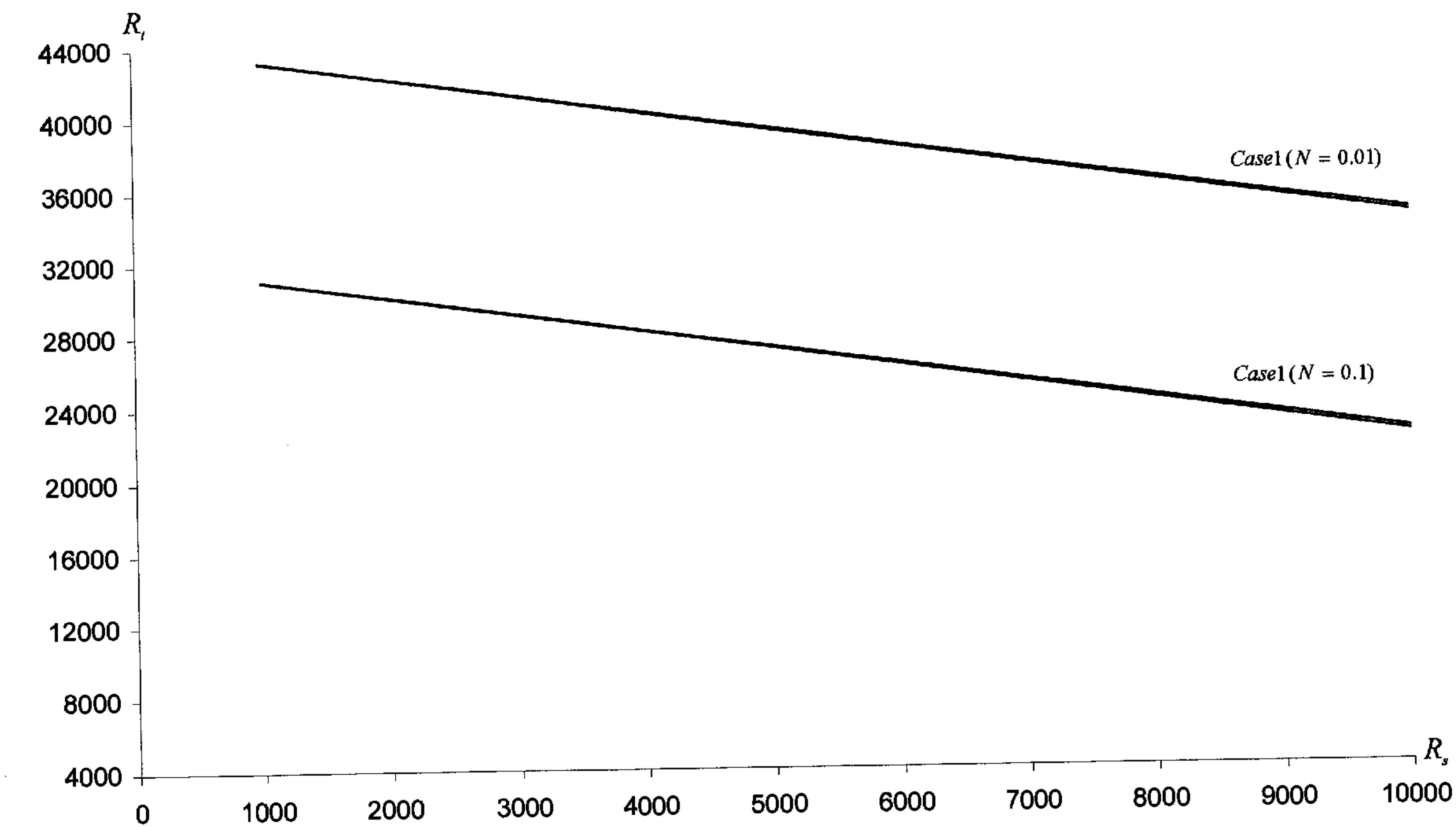


Figure 11: The relation between R_s and R_t for the overstability case when both boundaries are rigid for $N = 0.1, 0.01$, Q=5000 , $P_r = P_r' = 1$ and $P_m = 4$.

Table 21: The relation between Q and R_t for the stationary case when both boundaries are rigid for $N = 0.01$ and R_s =3000.

	Case 1		Case 2	
Q	a	R_t	a	R_t
300	3.6603	7597.5636	3.9601	8192.2915
450	4.0597	10248.4713	4.2713	10725.7163
600	4.1803	12738.6693	4.5168	13125.2037
750	4.5848	15070.8850	4.7209	15433.7927
900	4.7801	17346.1101	4.8961	17675.4738
1050	4.9490	19561.7131	5.0503	19865.2269
1200	5.0982	21730.3283	5.1115	22018.1818
1350	5.2291	23860.6519	5.3134	24126.4117
1500	5.3541	25958.9417	5.4278	26210.3534
1650	5.4659	28029.9414	5.5340	28269.0508
1800	5.5695	30077.2961	5.6326	30305.712
1950	5.6659	32103.9035	5.7246	32322.9134
2100	5.7563	34112.1081	5.8113	34322.7614
2250	5.8413	36103.8394	5.8930	36307.0047
2400	5.9218	38080.7079	5.9703	38277.1136
2550	5.9980	40044.0749	6.0418	40234.3442
2700	6.0706	41995.1034	6.1144	42179.7555
2850	6.1398	43934.7961	6.1807	44114.2852
3000	6.2060	45864.0255	6.1672	46044.9365

Table 22: The relation between Q and R_t for the overstability case when both boundaries are rigid for $N = 0.01$, R_s=3000 , $P_r = P_r' = 1$ and $P_m = 4$.

	Case 1		Case 2	
Q	a	R_t	a	R_t
2000	5.6557	24521.8078	5.6367	24437.8035
2500	5.8937	27622.5998	5.8787	27545.7707
3000	6.1002	30563.1598	6.0879	30492.2400
3500	6.2831	33378.5545	6.2727	33312.6272
4000	6.4476	36092.4817	6.4386	36030.8454
4500	6.5974	38721.8739	6.5895	38663.9737
5000	6.7349	41279.3433	6.7280	41224.7290
5500	6.8623	43774.5929	6.8560	43722.8941
6000	6.9810	46215.2828	6.9755	46166.1899
6500	7.0923	48607.5900	7.0873	48560.8412
7000	7.1970	50956.5843	7.1925	50911.9565
7500	7.2960	53266.4894	7.2918	53223.7894
8000	7.3900	55540.8697	7.3864	55499.9311
8500	7.4794	57782.7671	7.4754	57743.4444
9000	7.5648	59994.8028	7.5615	59956.9670
9500	7.6464	62179.2554	7.6435	62142.7936
10000	7.7247	64338.1207	7.7218	64302.9323
10500	7.8000	66473.1590	7.7973	66439.1538
11000	7.8723	68585.9328	7.8699	68553.0313

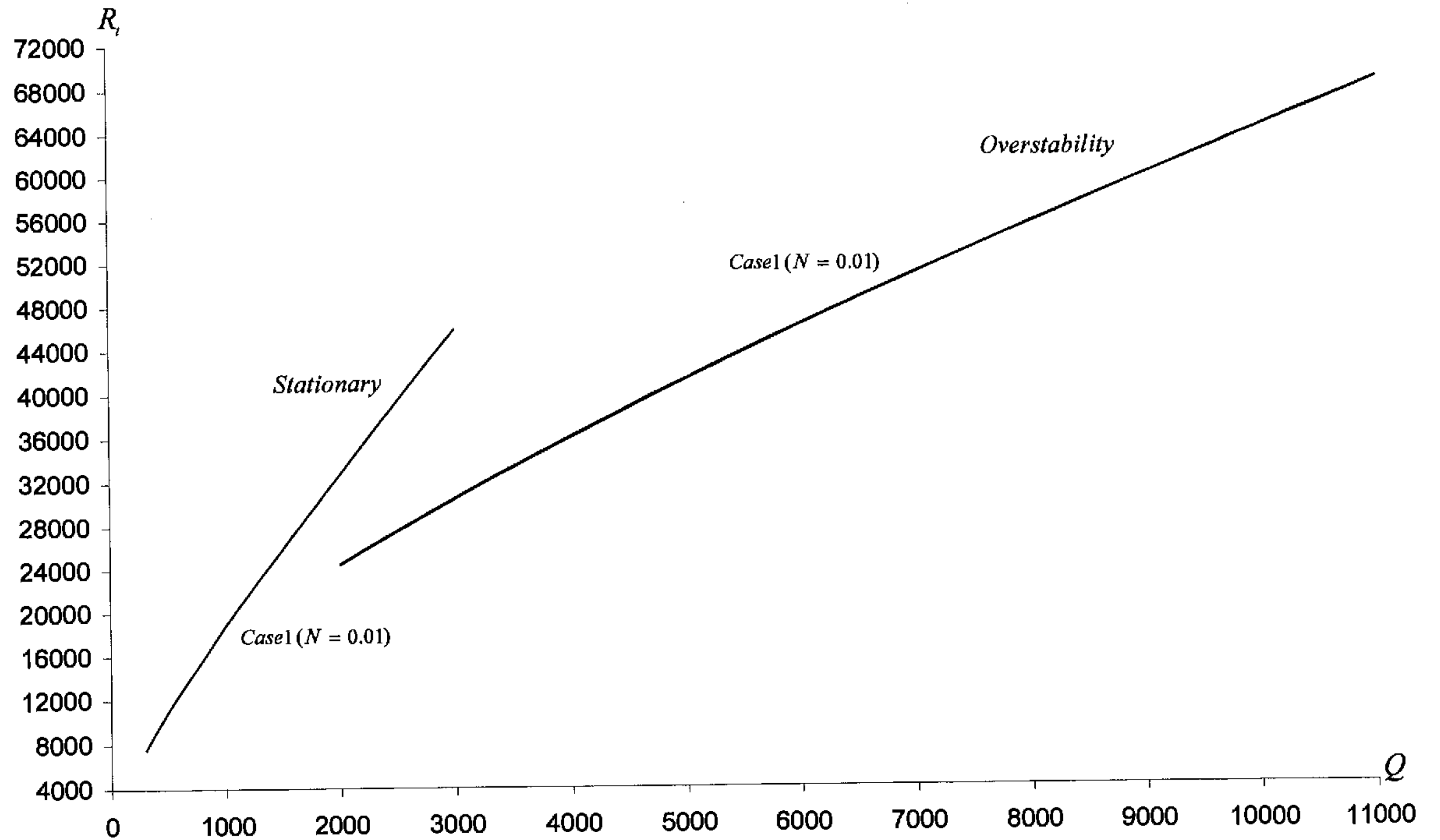


Figure 12: The relation between Q and R_t for the stationary and overstability cases when both boundaries are rigid for $N = 0.01$ and $R_s = 3000$.

Table23: The relation between Q and R_s for the stationary case when both boundaries are rigid for $N = 0.01$ and $R_t = 3000$.

	Case 1		Case 2	
Q	a	R_s	a	R_s
300	2.5614	5864.7121	3.5975	7341.8969
450	2.6513	7836.5922	3.8221	9609.7591
600	2.7364	9777.7492	4.0099	11801.9848
750	2.8183	11696.3945	4.1727	13939.1893
900	2.8977	13597.4046	4.3171	16033.8581
1050	2.9750	15483.9580	4.4474	18094.2688
1200	3.0507	17358.2661	4.5664	20126.2617
1350	3.1247	19221.9454	4.6762	22134.1496
1500	3.1973	21076.2232	4.7782	24121.2299
1650	3.2686	22922.0610	4.8736	26090.0946
1800	3.3384	24760.2314	4.9632	28042.8268
1950	3.4070	26591.3688	5.0478	29981.1320
2100	3.4744	28416.0041	5.1282	31906.4279
2250	3.5405	30234.5886	5.2042	33819.9106
2400	3.6053	32047.5114	5.2771	35722.5976
2550	3.6690	33855.1122	5.3467	37615.3663
2700	3.7314	35657.6903	5.4134	39498.9782
2850	3.7927	37455.5125	5.4774	41374.1001
3000	3.8528	39248.8184	5.5390	43241.3197

Table 24: The relation between Q and R_s for the overstability case when both boundaries are rigid for $N = 0.01$, $R_t = 3000$, $P_r = P_r' = 1$ and $P_m = 4$.

	Case 1		Case 2	
Q	a	R_s	a	R_s
2000	5.8992	25615.2418	5.7758	25055.3577
2500	6.1321	28827.0637	6.0088	28202.5783
3000	6.3300	31844.9242	6.2094	31175.1338
3500	6.5030	34715.8155	6.3863	34013.5780
4000	6.6574	37470.4522	6.5453	36744.6186
4500	6.7972	40130.1560	6.6898	39386.9906
5000	6.9265	42710.4174	6.8227	41954.4552
5500	7.0439	45222.8865	6.9457	44457.4834
6000	7.1543	47676.6001	7.0605	46904.2680
6500	7.2578	50078.7150	7.1681	49301.3670
7000	7.3554	52435.0212	7.2695	51654.1292
7500	7.4478	54750.2799	7.3655	53966.9873
8000	7.5355	57028.4627	7.4567	56243.6648
8500	7.6191	59272.9243	7.5435	58487.3267
9000	7.6991	61486.5292	7.6264	60700.6909
9500	7.7757	63671.7470	7.7058	62886.1125
10000	7.8493	65830.7252	7.7819	65045.6487
10500	7.9201	67965.3452	7.8552	67181.1097
11000	7.9883	70077.2672	7.9257	69294.0981

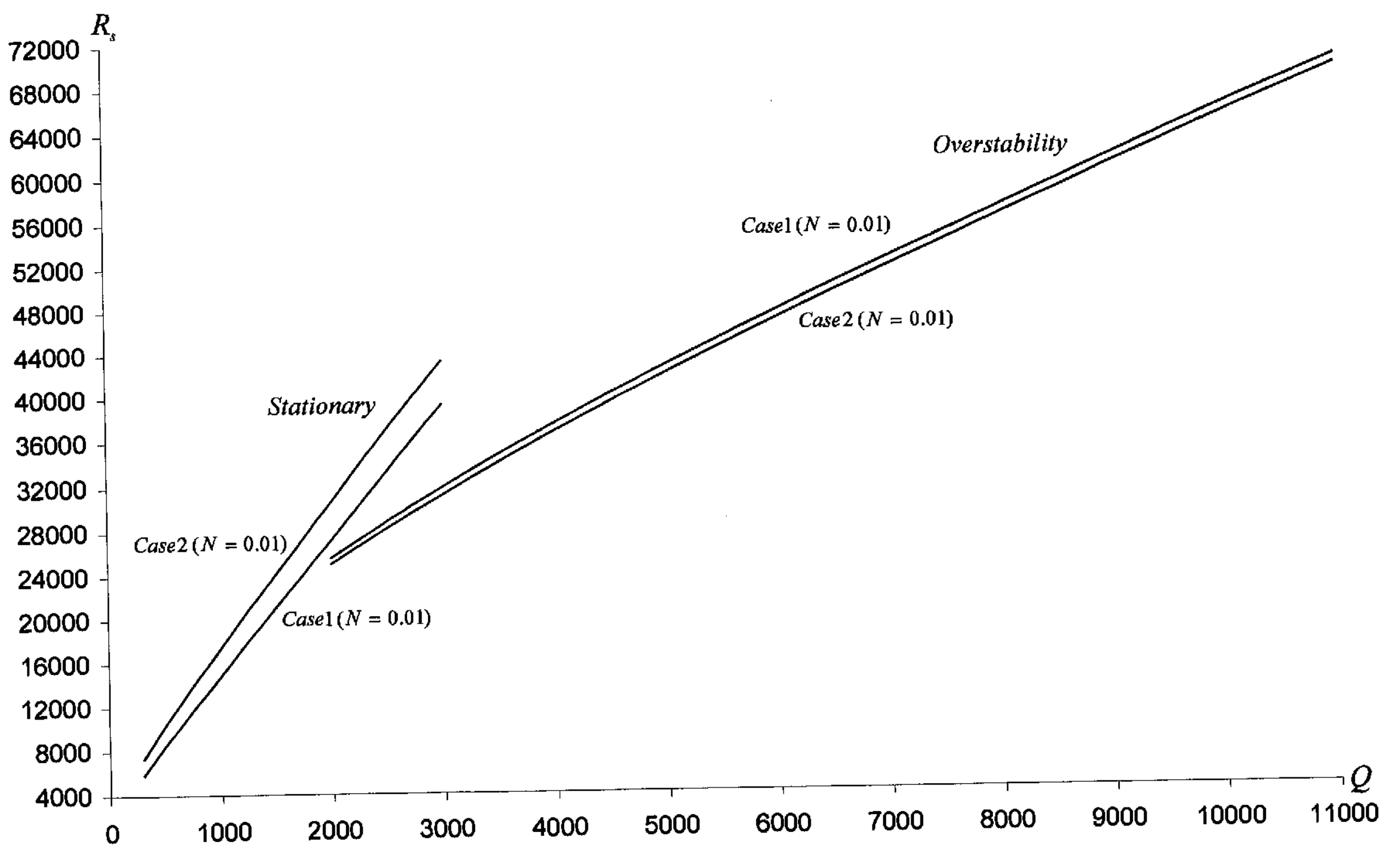


Figure 13: The relation between Q and R_s for the stationary and overstability cases when both boundaries are rigid for $N = 0.01$ and $R_t = 3000$.

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APPENDIX 1

```
      PROGRAMME BENARD
***  -----------------------------------------------------------------

***  THERMAL CONVECTION IN A HORIZONTAL POROUS LAYER AFFECTED
***  BY A VERTICAL MAGNETIC FIELD WHEN THE RELATION SHIP BETWEEN THE
***  MAGNETIC FIELD & THE MAGNETIC INDUCTION IS LINEAR.THE BOUNDARY
***  CONDITIONS ARE FREE (w=fi=theta=0), Db=0     AT Z=0 , Z=1.
***  THE FLUID LAYER IS HEATED FROM BELOW.
***  IN THIS PROGRAMME APPROXIMATION OF CHEBYSHEV POLYNOMIALS IS USED
***  TO SOLVE THE PROBLEM .


***  ---------------------------------------------------------------

**************************{ MAIN PROGRAMME}

***  ----------------------------------------------------------------

     IMPLICIT DOUBLE PRECISION (A-H,O-Z)
     PARAMETER   ( E = 1.0D-9 )
     COMMON N,R1,R2,PRV,PRM,Q
     OPEN  (8,FILE='C:\NAGFL15D\BATCH\AAMAL\RESULTA')
     WRITE(*,*) ' ENTER N,A,B,R1,R2,PRV,PRM,Q'
****  ----------------------------------------------------------------

***  WHERE :(N)    IS THE NUMBER OF POLYNOMIALS.
***********(A & B)    ARE TWO VALUES FOR THE WAVE NUMBER.
***********(R1 & R2) ARE TWO VALUES FOR THE RAYLEIGH NUMBER.
***********(PRV & PRM) BE RESPECTIVELY THE VISCOUS PRANDTLE NUMBER AND
****************THE MAGNETIC PRANDTLE NUMBER.
***********(Q)    IS CHANDRASEKHAR NUMBER.

****  ----------------------------------------------------------------

     READ*,N,A,B,R1,R2,PRV,PRM,Q
     RGOLD=(DSQRT(5.0D0)-1.0D0)/2.0D0
     M=DABS(DLOG(E/(B-A))/(DLOG(RGOLD)))
     A1=A+RGOLD*(B-A)

****  ----------------------------------------------------------------
     CALL CHEB(A1,U,G1R,G1I)
****  ----------------------------------------------------------------

     A2=A+RGOLD**2*(B-A)
```

```
**  ---------------------------------------------------------------
   CALL CHEB(A2,V,G2R,G2I)
**  ---------------------------------------------------------------

   DO 1 I=1,M
     IF (U.GT.V) THEN
      U=V
      B=A1
      A1=A2
      A2=A+RGOLD**2*(B-A)
**  ---------------------------------------------------------------
      CALL CHEB(A2,V,G2R,G2I)
**  ---------------------------------------------------------------

      ELSE
       V=U
       A=A2
       A2=A1
       A1=A+RGOLD*(B-A)
***  --------------------------------------------------------------
      CALL CHEB(A1,U,G1R,G1I)
***  --------------------------------------------------------------

      END IF
      IF ( DABS(U-V).LE.E) GO TO 50
  1  CONTINUE
 50  AMIN=(A1+A2)/2.0D0
***  --------------------------------------------------------------
        CALL CHEB(AMIN,R,SEGMAR,SEGMAI)
***  --------------------------------------------------------------

     PRINT*,'THE CRITICAL VALUE  OF THE  WAVE  NUMBER=',AMIN
     PRINT*,'THE THERMAL RAYLEIGH NUMBER=',R
     PRINT*,'THE REAL PART OF EIGENVALUE=',SEGMAR
     PRINT*,'THE IMAGINARY PART OF EIGENVALUE=',SEGMAI
     STOP
     END
***  --------------------------------------------------------------
***      THIS SUBROUTINE IS USED TO EVALUATE THE RAYLEIGH NUMBER
***  --------------------------------------------------------------

     SUBROUTINE CHEB(A1,R,SEGMAR,SEGMAI)
     IMPLICIT DOUBLE PRECISION (A-H,O-Z)
     COMMON N,R1,R2,PRV,PRM,Q
     PARAMETER (L1=30, L=4*L1)
```

```
      DIMENSION A(L,L),B(L,L),XI(L1,L1),D(L1,L1),D2(L1,L1),V(L1,L1),
     *          VD(L1,L1),Z(L,L),ITER(L),ALFR(L),ALFI(L),BETA(L),GR(L),
     *          GI(L),ELANDAR(L),ELANDAI(L),ST(100)
      LOGICAL   MATZ
      EXTERNAL  X02AAF
      EXTERNAL  F02BJF
      ST(1)=R1
      ST(2)=R2
      NL1=N+1
      NL2=N+2
      NL3=N+3
      N2=2*N
      N2L1=N2+1
      N2L2=N2+2
      N2L3=N2+3
      N3=3*N
      N3L1=N3+1
      N3L2=N3+2
      N3L3=N3+3
      N4=4*N

*** ------------------------------------------------------------------
***************************** THE IDENTITY MATRIX   (XI)
*** ------------------------------------------------------------------

       DO 3 I=1,N
         DO 2 J=1,N
           IF(I.EQ.J) THEN
            XI(I,J)= 1.0D0
           ELSE
            XI(I,J)=0.0D0
           END IF
    2    CONTINUE
    3  CONTINUE

*** ------------------------------------------------------------------
       CALL DERIVE (N,D)
       CALL PRO(N,D,D,D2)
*** ------------------------------------------------------------------

       DO 5 I=1,N
         DO 4 J=1,N
           V(I,J)=4.0D0*(D2(I,J)-((A1/2.0D0)**2*XI(I,J)))
    4    CONTINUE

*** ------------------------------------------------------------------
********************************** THE SECANT METHOD
*** ------------------------------------------------------------------

       DO 33 K=1,100
         IF (K.GE.3) THEN
            ST(K)=(ST(K-2)*GR(K-1)-ST(K-1)*GR(K-2))/(GR(K-1)-GR(K-2))
```

```
      END IF
***  ---------------------------------------------------------------------
******************************* THE A & B ZEROING MATRIX
***  ---------------------------------------------------------------------

      DO 7 I=1,N4
        DO 6 J=1,N4
          A(I,J)=0.0D0
          B(I,J)=0.0D0
        CONTINUE
      CONTINUE
***  ---------------------------------------------------------------------
******************************* THE ELEMENTS [ A11 & B11 ]
***  ---------------------------------------------------------------------

      DO 9 I=3,N
        DO 8 J=1,N
          A(1,J)=1.0D0
          A(2,J)=(-1.0D0)**(J+1)
          A(I,J)=V(I-2,J)
          B(I,J)=XI(I-2,J)
8         CONTINUE
9     CONTINUE
***  ---------------------------------------------------------------------
******************************* THE ELEMENT [ A22 & B22 ]
***  ---------------------------------------------------------------------

      DO 11 I=NL3,N2
        DO 10 J=NL1,N2
          A(NL1,J)=1.0D0
          A(NL2,J)=(-1.0D0)**(J-NL1)
          A(I,J)   =V(I-NL2,J-N)
          B(I,J)= PRV* XI(I-NL2,J-N)
0         CONTINUE
1     CONTINUE
***  ---------------------------------------------------------------------
******************************* THE ELEMENT [ B33 ]
***  ---------------------------------------------------------------------

      DO 13 I=N2L3 , N3
        DO 12 J=N2L1 , N3
         B(I,J)=PRM*XI(I-N2L2,J-N2)
12      CONTINUE
13    CONTINUE
***  ---------------------------------------------------------------------
******************************* THE ELEMENT [ A33 ]
***  ---------------------------------------------------------------------
```

```
      DO 14 I=1,N
        A(N2L1,I+N2)= 2.0D0 * DBLE ((I-1)**2)
        A(N2L2,I+N2)= 2.0D0 * DBLE ((I-1)**2)*(-1.0D0)**(I)
      CONTINUE
      DO 16 I=N2L3,N3
        DO 15 J=N2L1,N3
          A(I,J)= V(I-N2L2,J-N2)
5       CONTINUE
5     CONTINUE

**** ---------------------------------------------------------------
******************************** THE ELEMENTS [ A44 ]
**** ---------------------------------------------------------------

      DO 18 I=N3L3,N4
        DO 17 J=N3L1,N4
          A(N3L1,J)=1.0D0
          A(N3L2,J)=(-1.0D0)**(J-NL1)
          A(I,J)= V(I-N3L2,J-N3)
7       CONTINUE
8     CONTINUE

**** ---------------------------------------------------------------
******************************** THE ELEMENT [A12]
**** ---------------------------------------------------------------

      DO 20 I=3,N
        DO 19 J=NL1,N2
          A(I,J)=(-1.0D0)* DSQRT(ST(K))*(A1**2)*XI(I-2,J-N)
9       CONTINUE
0     CONTINUE

**** ---------------------------------------------------------------
******************************** THE ELEMENT [ A14 ]
**** ---------------------------------------------------------------

      DO 22 I=3,N
        DO 21 J=N3L1,N4
          A(I,J)= (-1.0D0)*(4.0D0)*Q*D2(I-2,J-N3)
1       CONTINUE
2     CONTINUE

**** ---------------------------------------------------------------
******************************** THE ELEMENT [ A24 ]
**** ---------------------------------------------------------------

      DO 24 I=NL3,N2
        DO 23 J=N3L1,N4
          A(I,J)=DSQRT(ST(K))*XI(I-NL2,J-N3)
23      CONTINUE
```

```
4    CONTINUE

** -----------------------------------------------------------------
********************************** THE ELEMENT [ A34 ]
** -----------------------------------------------------------------

      DO 26 I=N2L3,N3
        DO 25 J=N3L1,N4
          A(I,J)=2.0D0*Q*D(I-N2L2,J-N3)
5       CONTINUE
6     CONTINUE

** -----------------------------------------------------------------
********************************* THE ELEMENT [ A41 ]
** -----------------------------------------------------------------

      DO 28 I=N3L3,N4
        DO 27 J=1,N
          A(I,J)=(-1.0D0)*XI(I-N3L2,J)
7       CONTINUE
8     CONTINUE


** -----------------------------------------------------------------
********************************** THE ELEMENT [ B13 ]
** -----------------------------------------------------------------

      DO 30 I=3,N
        DO 29 J=N2L1,N3
          B(I,J)=(-1.0D0)*2.0D0*PRM*D(I-2,J-N2)
29      CONTINUE
30    CONTINUE

*** ----------------------------------------------------------------


********************************** WE WILL USE NAGROUTINE (F02BJF)

      MATZ=.FALSE.
      EPS1=X02AAF(0.0D0)
      IFAIL=0

*** ----------------------------------------------------------------
      CALL F02BJF ( N4,A,L,B,L,EPS1,ALFR,ALFI,BETA,MATZ,Z,L,ITER,
     *              IFAIL)
**** ---------------------------------------------------------------

      J=0
      DO 31 I=1,N4
        IF ( BETA(I).NE. 0.0D0) THEN
          J=1+J
          ELANDAR(J)=ALFR(I)/BETA(I)
```

```
        ELANDAI(J)=ALFI(I)/BETA(I)
      END IF
    CONTINUE
    ELARGER=ELANDAR(1)
    DO 32 I=2,J
      IF ( ELANDAR(I).GT.ELARGER) THEN
        ELARGER=ELANDAR(I)
        ELARGEI=ELANDAI(I)
      END IF
    CONTINUE
    GR(K)=ELARGER
    GI(K)=ELARGEI
    IF (K.LT.2) GO TO 33
        IF( DABS(ST(K)-ST(K-1)).LE.1.0D-7)THEN
          R=ST(K)
          SEGMAR=GR(K)
          SEGMAI=GI(K)
         RETURN
     END IF
  CONTINUE
  END

** ------------------------------------------------------------------

*********THIS SUBROUTINE CALCULATES THE PRODUCT OF TWO MATRICES

** ------------------------------------------------------------------

   SUBROUTINE PRO (N,H,T,Z)
   IMPLICIT DOUBLE PRECISION (A-H,O-Z)
   PARAMETER (L1=30)
   DIMENSION H(L1,L1), T(L1,L1), Z(L1,L1)
   DO 35 I=1,N
     DO 35 J=1,N
       Z(I,J)=0.0D0
       DO 34 K=1,N
         Z(I,J)=Z(I,J)+H(I,K)*T(K,J)
4      CONTINUE
5  CONTINUE
   RETURN
   END
*** ------------------------------------------------------------------

***************THIS SUBROUTINE BUILDS THE DERIVATIVE MATRIX

*** ------------------------------------------------------------------

   SUBROUTINE DERIVE (N,D)
   IMPLICIT DOUBLE PRECISION (A-H,O-Z)
   PARAMETER (L1=30)
   DIMENSION D(L1,L1)
```

```
      DO 36 I=1,N
        DO 36 J=1,N
          IF((J.GT.I) .AND. (MOD(J-I,2) .EQ. 1)) THEN
          D(I,J)=2.0D0*DBLE(J-1)
         ELSE
          D(I,J)=0.0D0
          END IF
6     CONTINUE
      DO 37 I=1,N
        D(1,I)=0.5D0*D(1,I)
7     CONTINUE
      RETURN
      END
```

APPENDIX 2

```
   PROGRAMME FREE 1
** ---------------------------------------------------------------------

** THERMOSOLUTAL CONVECTION IN A HORIZONTAL POROUS LAYER AFFECTED
** BY A VERTICAL MAGNETIC FIELD WHEN THE RELATION SHIP BETWEEN THE
** MAGNETIC FIELD & THE MAGNETIC INDUCTION IS LINEAR, THE
** BOUNDARY CONDITIONS ARE FREE
**************(w=fi=theta=0), Db=Dc=0     AT Z=0 , Z=1.     (CASE 1)
** THE FLUID LAYER IS HEATED FROM BELOW & SOLUTED FROM ABOVE.
** IN THIS PROGRAMME APPROXIMATION OF CHEBYSHEV POLYNOMIALS IS USED
** TO SOLVE THE PROBLEM .

** --------------------------------------------------------------------

*********************** { MAIN PROGRAMME}

*** -------------------------------------------------------------------

   IMPLICIT DOUBLE PRECISION (A-H,O-Z)
   PARAMETER  ( E = 1.0D-9 )
   COMMON N,R1,R2,PRV,PRS,PRM,NP,Q,SS,SSS
   OPEN (8,FILE='C:\NAGFL15D\BATCH\AAMAL\RESULT1')
   WRITE(*,*) ' ENTER N,A,B,R1,R2,PRV,PRS,PRM,NP,Q,SS,SSS'
*** ---------------------------------------------------------------------

*** WHERE :(N)    IS THE NUMBER OF POLYNOMIALS.
********* (A & B)   ARE TWO VALUES FOR THE WAVE NUMBER.
********* (R1 & R2) ARE TWO VALUES FOR THE RAYLEIGH NUMBER.
******* (PRV & PRS & PRM) BE RESPECTIVELY THE VISCOUS PRANDTLE NUMBER, THE
************** SOLUTE PRANDTLE NUMBER AND THE MAGNETIC PRANDTLE NUMBER.
********* (NP)   IS THE INVERSE OF THE NON-DIMENSIONAL PERMEABILITY.
********* (Q)    IS CHANDRASEKHAR NUMBER.
********* (SS)   IS THE PARAMETER WHICH DECIDES THE REQUIRD RAYLEIGH
*************** NUMBER THERMAL OR SOLUTE.
********* (SSS) IS THE VALUE OF THE KNOWN RAYLEIGH NUMBER.

*** -------------------------------------------------------------------

   READ*,N,A,B,R1,R2,PRV,PRS,PRM,NP,Q,SS,SSS
   RGOLD=(DSQRT(5.0D0)-1.0D0)/2.0D0
   M=DABS(DLOG(E/(B-A))/(DLOG(RGOLD)))
   A1=A+RGOLD*(B-A)

*** -------------------------------------------------------------------
   CALL CHEB(A1,U,G1R,G1I)
*** -------------------------------------------------------------------

   A2=A+RGOLD**2*(B-A)
```

```
** ----------------------------------------------------------------
  CALL CHEB(A2,V,G2R,G2I)
** ----------------------------------------------------------------

  DO 1 I=1,M
    IF (U.GT.V) THEN
     U=V
     B=A1
     A1=A2
     A2=A+RGOLD**2*(B-A)

** ----------------------------------------------------------------
    CALL CHEB(A2,V,G2R,G2I)
** ----------------------------------------------------------------

    ELSE
     V=U
     A=A2
     A2=A1
     A1=A+RGOLD*(B-A)

** ----------------------------------------------------------------
    CALL CHEB(A1,U,G1R,G1I)
** ----------------------------------------------------------------

    END IF
    IF ( DABS(U-V).LE.E) GO TO 50
   CONTINUE
0  AMIN=(A1+A2)/2.0D0

*** ---------------------------------------------------------------
      CALL CHEB(AMIN,R,SEGMAR,SEGMAI)
*** ---------------------------------------------------------------

   PRINT*,'THE CRITICAL VALUE  OF THE  WAVE  NUMBER=',AMIN
   IF(SS.EQ.0.0D0) THEN
     PRINT*,'THE THERMAL RAYLEIGH NUMBER=',R
     PRINT*,'THE SOLUTE  RAYLEIGH NUMBER=',SSS
   ELSE
     PRINT*,'THE SOLUTE  RAYLEIGH NUMBER=',R
     PRINT*,'THE THERMAL RAYLEIGH NUMBER=',SSS
   END IF
   PRINT*,'THE REAL PART OF EIGENVALUE=',SEGMAR
   PRINT*,'THE IMAGINARY PART OF EIGENVALUE=',SEGMAI
   STOP
   END

*** ---------------------------------------------------------------
***     THIS SUBROUTINE IS USED TO EVALUATE THE RAYLEIGH NUMBER
*** ---------------------------------------------------------------
   SUBROUTINE CHEB(A1,R,SEGMAR,SEGMAI)
   IMPLICIT DOUBLE PRECISION (A-H,O-Z)
```

```
      COMMON N,R1,R2,PRV,PRS,PRM,NP,Q,SS,SSS
      PARAMETER (L1=30, L=5*L1)
      DIMENSION A(L,L),B(L,L),XI(L1,L1),D(L1,L1),D2(L1,L1),V(L1,L1),
     *          VD(L1,L1),Z(L,L),ITER(L),ALFR(L),ALFI(L),BETA(L),GR(L),
     *          GI(L),ELANDAR(L),ELANDAI(L),ST(100),SW(100),VNP(L1,L1)
      LOGICAL   MATZ
      EXTERNAL  X02AAF
      EXTERNAL  F02BJF
      IF(SS.EQ.0.0D0)THEN
        ST(1)=R1
        ST(2)=R2
      ELSE
        SW(1)=R1
        SW(2)=R2
      END IF
      NL1=N+1
      NL2=N+2
      NL3=N+3
      N2=2*N
      N2L1=N2+1
      N2L2=N2+2
      N2L3=N2+3
      N3=3*N
      N3L1=N3+1
      N3L2=N3+2
      N3L3=N3+3
      N4=4*N
      N4L1=N4+1
      N4L2=N4+2
      N4L3=N4+3
      N5=5*N
***  ---------------------------------------------------------------
*************************** THE IDENTITY MATRIX  (XI)
***  ---------------------------------------------------------------

      DO 3 I=1,N
        DO 2 J=1,N
          IF(I.EQ.J) THEN
           XI(I,J)= 1.0D0
          ELSE
           XI(I,J)=0.0D0
          END IF
    2   CONTINUE
    3 CONTINUE

***  ---------------------------------------------------------------
      CALL DERIVE (N,D)
      CALL PRO(N,D,D,D2)
***  ---------------------------------------------------------------
```

```
      DO 5 I=1,N
        DO 4 J=1,N
          V(I,J)=4.0D0*(D2(I,J)-((A1/2.0D0)**2*XI(I,J)))
        CONTINUE
***  -------------------------------------------------------------------
      CALL PRO (N,V,D,VD)
***  -------------------------------------------------------------------
      DO 7 I=1,N
        DO 6 J=1,N
          VNP(I,J)=V(I,J)-NP*XI(I,J)
        CONTINUE
      CONTINUE

***  -------------------------------------------------------------------
**************************  THE SECANT METHOD
***  -------------------------------------------------------------------


      DO 40 K=1,100
        IF (SS.EQ.0.0D0) THEN
          SW(K)=SSS
          IF (K.GE.3) THEN
            ST(K)=(ST(K-2)*GR(K-1)-ST(K-1)*GR(K-2))/(GR(K-1)-GR(K-2))
        END IF
        ELSE
          ST(K)=SSS
          IF (K.GE.3) THEN
            SW(K)=(SW(K-2)*GR(K-1)-SW(K-1)*GR(K-2))/(GR(K-1)-GR(K-2))
          END IF
        END IF
***  -------------------------------------------------------------------
***************************** THE A & B ZEROING MATRIX
***  -------------------------------------------------------------------

        DO 9 I=1,N5
          DO 8 J=1,N5
            A(I,J)=0.0D0
            B(I,J)=0.0D0
          CONTINUE
        CONTINUE

***  -------------------------------------------------------------------
***************************** THE ELEMENTS [ A11 & B11 ]
***  -------------------------------------------------------------------

        DO 11 I=3,N
          DO 10 J=1,N
            A(1,J)=1.0D0
            A(2,J)=(-1.0D0)**(J+1)
            A(I,J)=VNP(I-2,J)
            B(I,J)=XI(I-2,J)
```

```
        CONTINUE
      CONTINUE

**  -----------------------------------------------------------------
****************************** THE ELEMENT [ A22 & B22 ]
**  -----------------------------------------------------------------

      DO 13 I=NL3,N2
        DO 12 J=NL1,N2
          A(NL1,J)=1.0D0
          A(NL2,J)=(-1.0D0)**(J-NL1)
          A(I,J)  =V(I-NL2,J-N)
          B(I,J)= PRV* XI(I-NL2,J-N)
        CONTINUE
      CONTINUE

**  -----------------------------------------------------------------
****************************** THE ELEMENT [ B33 ]
**  -----------------------------------------------------------------

      DO 15 I=N2L3 , N3
        DO 14 J=N2L1 , N3
         B(I,J)=PRM*XI(I-N2L2,J-N2)
14      CONTINUE
15    CONTINUE

**  -----------------------------------------------------------------
****************************** THE ELEMENT [ A33 ]
**  -----------------------------------------------------------------

      DO 16 I=1,N
        A(N2L1,I+N2)= 2.0D0 * DBLE ((I-1)**2)
        A(N2L2,I+N2)= 2.0D0 * DBLE ((I-1)**2)*(-1.0D0)**(I)
6     CONTINUE

      DO 18 I=N2L3,N3
        DO 17 J=N2L1,N3
          A(I,J)= V(I-N2L2,J-N2)
7       CONTINUE
8     CONTINUE


***  ----------------------------------------------------------------
****************************** THE ELEMENTS [ A44 , B44 ]
***  ----------------------------------------------------------------

      DO 19 I=1,N
        A(N3L1,I+N3)= 2.0D0 * DBLE ((I-1)**2)
        A(N3L2,I+N3)= 2.0D0*DBLE ((I-1)**2)*(-1.0D0)**(I)
9     CONTINUE
      DO 21 I=N3L3,N4
        DO 20 J=N3L1,N4
```

```
        A(I,J)= V(I-N3L2,J-N3)
        B(I,J)=PRS * XI(I-N3L2,J-N3)
      CONTINUE
    CONTINUE

** --------------------------------------------------------------------
********************************* THE ELEMENT [ A55 ]
** --------------------------------------------------------------------

    DO 23 I=N4L3,N5
      DO 22 J=N4L1,N5
        A(N4L1,J)= 1.0D0
        A(N4L2,J)= (-1.0D0)**(J-N4L1)
        A(I,J)    = V(I-N4L2,J-N4)
      CONTINUE
    CONTINUE

** --------------------------------------------------------------------
********************************* THE ELEMENT [A12]
** --------------------------------------------------------------------

    DO 25 I=3,N
      DO 24 J=NL1,N2
        A(I,J)=(-1.0D0)* DSQRT(ST(K))*(A1**2)*XI(I-2,J-N)
4     CONTINUE
5   CONTINUE

*** -------------------------------------------------------------------
********************************* THE ELEMENT [A13]
*** -------------------------------------------------------------------

    DO 27 I=3,N
      DO 26 J=N2L1,N3
        A(I,J)=2.0D0*VD(I-2,J-N2)
26    CONTINUE
27  CONTINUE

*** -------------------------------------------------------------------
********************************* THE ELEMENT [ A14 ]
*** -------------------------------------------------------------------
    DO 29 I=3,N
      DO 28 J=N3L1,N4
        A(I,J)= DSQRT(SW(K))*(A1**2)*XI(I-2,J-N3)
8     CONTINUE
9   CONTINUE

*** -------------------------------------------------------------------
********************************* THE ELEMENT [ A25 ]
*** -------------------------------------------------------------------
```

```
      DO 31 I=NL3,N2
        DO 30 J=N4L1,N5
          A(I,J)=DSQRT(ST(K))*XI(I-NL2,J-N4)
        CONTINUE
      CONTINUE

*  -------------------------------------------------------------------
******************************* THE ELEMENT [A35]
**  ------------------------------------------------------------------

      DO 33 I=N2L3,N3
        DO 32 J=N4L1,N5
          A(I,J)=2.0D0*Q*D(I-N2L2,J-N4)
2       CONTINUE
3     CONTINUE

**  ------------------------------------------------------------------
******************************* THE ELEMENT [ A45 ]
**  ------------------------------------------------------------------

      DO 35 I=N3L3,N4
        DO 34 J=N4L1,N5
          A(I,J)=(-1.0D0)*DSQRT(SW(K))*XI(I-N3L2,J-N4)
        CONTINUE
      CONTINUE

**  ------------------------------------------------------------------
******************************  THE ELEMENT [ A51 ]
**  ------------------------------------------------------------------

      DO 37 I=N4L3,N5
        DO 36 J=1,N
          A(I,J)=(-1.0D0)*XI(I-N4L2,J)
5       CONTINUE
7     CONTINUE

***  -----------------------------------------------------------------

************************ WE WILL USE NAGROUTINE (F02BJF)

      MATZ=.FALSE.
      EPS1=X02AAF(0.0D0)
      IFAIL=0

***  -----------------------------------------------------------------
      CALL F02BJF ( N5,A,L,B,L,EPS1,ALFR,ALFI,BETA,MATZ,Z,L,ITER,
     *              IFAIL)
***  -----------------------------------------------------------------

      J=0
      DO 38 I=1,N5
        IF ( BETA(I).NE. 0.0D0) THEN
```

```
        J=1+J
        ELANDAR(J)=ALFR(I)/BETA(I)
        ELANDAI(J)=ALFI(I)/BETA(I)
      END IF
    CONTINUE
    ELARGER=ELANDAR(1)
    DO 39 I=2,J
    IF ( ELANDAR(I).GT.ELARGER) THEN
      ELARGER=ELANDAR(I)
      ELARGEI=ELANDAI(I)
    END IF
    CONTINUE
    GR(K)=ELARGER
    GI(K)=ELARGEI
    IF ( K.LT.2) GO TO 40
      IF(SS.EQ.0.0D0) THEN
        IF( DABS(ST(K)-ST(K-1)).LE.1.0D-7)THEN
          R=ST(K)
          SEGMAR=GR(K)
          SEGMAI=GI(K)
          RETURN
        END IF
        ELSE
        IF(DABS(SW(K)-SW(K-1)).LE.1.0D-7)THEN
          R=SW(K)
          SEGMAR=GR(K)
          SEGMAI=GI(K)
          RETURN
        END IF
      END IF
    CONTINUE
    END
**  -----------------------------------------------------------------
*********** THIS SUBROUTINE CALCULATES THE PRODUCT OF TWO MATRICES
**  -----------------------------------------------------------------

    SUBROUTINE PRO (N,H,T,Z)
    IMPLICIT DOUBLE PRECISION (A-H,O-Z)
    PARAMETER (L1=30)
    DIMENSION H(L1,L1), T(L1,L1), Z(L1,L1)
    DO 42 I=1,N
      DO 42 J=1,N
        Z(I,J)=0.0D0
        DO 41 K=1,N
          Z(I,J)=Z(I,J)+H(I,K)*T(K,J)
1       CONTINUE
2   CONTINUE
    RETURN
    END
```

```
**  ------------------------------------------------------------------
*************** THIS SUBROUTINE BUILDS THE DERIVATIVE MATRIX
**  ------------------------------------------------------------------

      SUBROUTINE DERIVE (N,D)
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      PARAMETER (L1=30)
      DIMENSION D(L1,L1)
      DO 43 I=1,N
        DO 43 J=1,N
          IF((J.GT.I) .AND. (MOD(J-I,2) .EQ. 1)) THEN
          D(I,J)=2.0D0*DBLE(J-1)
          ELSE
          D(I,J)=0.0D0
          END IF
3     CONTINUE
      DO 44 I=1,N
        D(1,I)=0.5D0*D(1,I)
4     CONTINUE
      RETURN
      END
```

APPENDIX 3

```
    PROGRAME FREE 2
*** ---------------------------------------------------------------------

*** THERMOSOLUTAL CONVECTION IN A HORIZONTAL POROUS LAYER AFFECTED
*** BY A VERTICAL MAGNETIC FIELD WHEN THE RELATION SHIP BETWEEN THE
*** MAGNETIC FIELD & THE MAGNETIC INDUCTION IS LINEAR, THE BOUNDARY
*** CONDITIONS ARE FREE
******* (w=fi=theta=0), Db=0      AT Z=0 , Z=1.
******* C=0   AT   Z=0    &    DC=0   AT   Z=1.      (CASE 2)
*** THE FLUID LAYER IS HEATED FROM BELOW & SOLUTED FROM ABOVE.
*** IN THIS PROGRAMME APPROXIMATION OF CHEBYSHEV POLYNOMIALS IS USED
*** TO SOLVE THE PROBLEM.

*** -------------------------------------------------------------------

*************************** { MAIN PROGRAMME }

*** -------------------------------------------------------------------

   IMPLICIT DOUBLE PRECISION (A-H,O-Z)
   PARAMETER  ( E = 1.0D-9 )
   COMMON N,R1,R2,PRV,PRS,PRM,NP,Q,SS,SSS
   OPEN (8,FILE='C:\NAGFL15D\BATCH\AAMAL\RESULT2')
   WRITE(*,*) ' ENTER N,A,B,R1,R2,PR,PRP,PM,NP,Q,SS,SSS'

*** --------------------------------------------------------------------

*** WHERE : (N)     IS THE NUMBER OF POLYNOMIALS.
**********  (A & B)  ARE TWO VALUES FOR THE WAVE NUMBER.
**********  (PRV & PRS & PRM) BE RESPECTIVELY THE VISCOUS PRANDTLE
***************** NUMBER, THE SOLUTE PRANDTLE NUMBER AND THE MAGNETIC
***************** PRANDTLE NUMBER.
**********  (NP)    IS THE INVERSE OF THE NON-DIMENSIONAL PERMEABILITY.
**********  (Q)     IS CHANDRASEKHAR NUMBER.
**********  (SS)    IS THE PARAMETER WHICH DECIDES THE REQUIRD RAYLEIGH
***************** NUMBER THERMAL OR SOLUTE.
**********  (SSS)   IS THE VALUE OF THE KNOWN NUMBER.

*** ---------------------------------------------------------------------

   READ*,N,A,B,R1,R2,PRV,PRS,PRM,NP,Q,SS,SSS
   RGOLD=(DSQRT(5.0D0)-1.0D0)/2.0D0
   M=DABS(DLOG(E/(B-A))/(DLOG(RGOLD)))
   A1=A+RGOLD*(B-A)

*** ----------------------------------------------------------------------
   CALL CHEB(A1,U,G1R,G1I)
*** ----------------------------------------------------------------------
```

```
      A2=A+RGOLD**2*(B-A)

C*** -------------------------------------------------------------
      CALL CHEB(A2,V,G2R,G2I)
C*** -------------------------------------------------------------

      DO 1 I=1,M
        IF (U.GT.V) THEN
         U=V
         B=A1
         A1=A2
         A2=A+RGOLD**2*(B-A)

C*** -------------------------------------------------------------
        CALL CHEB(A2,V,G2R,G2I)
C*** -------------------------------------------------------------

        ELSE
         V=U
         A=A2
         A2=A1
         A1=A+RGOLD*(B-A)

C*** -------------------------------------------------------------
        CALL CHEB(A1,U,G1R,G1I)
C*** -------------------------------------------------------------

        END IF
        IF ( DABS(U-V).LE.E) GO TO 50
    1 CONTINUE
   50 AMIN=(A1+A2)/2.0D0

C*** -------------------------------------------------------------
        CALL CHEB(AMIN,R,SEGMAR,SEGMAI)
C*** -------------------------------------------------------------

       PRINT*,'THE CRITICAL VALUE  OF THE  WAVE  NUMBER=',AMIN
         IF(SS.EQ.0.0D0) THEN
           PRINT*,'THE THERMAL RAYLEIGH NUMBER=',R
           PRINT*,'THE SOLUTE  RAYLEIGH NUMBER=',SSS
          ELSE
           PRINT*,'THE SOLUTE  RAYLEIGH NUMBER=',R
           PRINT*,'THE THERMAL RAYLEIGH NUMBER=',SSS
         END IF
       PRINT*,'THE REAL PART OF EIGENVALUE=',SEGMAR
       PRINT*,'THE IMAGINARY PART OF EIGENVALUE=',SEGMAI
       STOP
       END
```

```
** ---------------------------------------------------------------------

******* THIS SUBROUTINE IS USED TO EVALUATE THE RAYLEIGH NUMBER

*** --------------------------------------------------------------------

      SUBROUTINE CHEB(A1,R,SEGMAR,SEGMAI)
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      COMMON N,R1,R2,PRV,PRS,PRM,NP,Q,SS,SSS
      PARAMETER (L1=30, L=5*L1)
      DIMENSION A(L,L),B(L,L),XI(L1,L1),D(L1,L1),D2(L1,L1),V(L1,L1),
     *          VD(L1,L1),Z(L,L),ITER(L),ALFR(L),ALFI(L),BETA(L),GR(L),
     *          GI(L),ELANDAR(L),ELANDAI(L),ST(100),SW(100),VNP(L1,L1)
      LOGICAL   MATZ
      EXTERNAL  X02AAF
      EXTERNAL  F02BJF
      IF(SS.EQ.0.0D0)THEN
        ST(1)=R1
        ST(2)=R2
      ELSE
        SW(1)=R1
        SW(2)=R2
      END IF
      NL1=N+1
      NL2=N+2
      NL3=N+3
      N2=2*N
      N2L1=N2+1
      N2L2=N2+2
      N2L3=N2+3
      N3=3*N
      N3L1=N3+1
      N3L2=N3+2
      N3L3=N3+3
      N4=4*N
      N4L1=N4+1
      N4L2=N4+2
      N4L3=N4+3
      N5=5*N

*** --------------------------------------------------------------------
********************** THE IDENTITY MATRIX  (XI)
*** --------------------------------------------------------------------

       DO 3 I=1,N
         DO 2 J=1,N
           IF(I.EQ.J) THEN
            XI(I,J)= 1.0D0
           ELSE
            XI(I,J)=0.0D0
           END IF
```

```
    CONTINUE
  CONTINUE

*** ------------------------------------------------------------------
  CALL DERIVE (N,D)
  CALL PRO(N,D,D,D2)
*** ------------------------------------------------------------------

  DO 5 I=1,N
    DO 4 J=1,N
      V(I,J)=4.0D0*(D2(I,J)-((A1/2.0D0)**2*XI(I,J)))
    CONTINUE

*** ------------------------------------------------------------------
  CALL PRO (N,V,D,VD)
*** ------------------------------------------------------------------

  DO 7 I=1,N
    DO 6 J=1,N
      VNP(I,J)=V(I,J)-NP*XI(I,J)
    CONTINUE
  CONTINUE

*** ------------------------------------------------------------------
************************* THE SECANT METHOD
*** ------------------------------------------------------------------

  DO 40 K=1,100
    IF (SS.EQ.0.0D0) THEN
      SW(K)=SSS
      IF (K.GE.3) THEN
        ST(K)=(ST(K-2)*GR(K-1)-ST(K-1)*GR(K-2))/(GR(K-1)-GR(K-2))
      END IF
    ELSE
      ST(K)=SSS
      IF (K.GE.3) THEN
        SW(K)=(SW(K-2)*GR(K-1)-SW(K-1)*GR(K-2))/(GR(K-1)-GR(K-2))
      END IF
    END IF

*** ------------------------------------------------------------------
*********************** THE A & B ZEROING MATRIX
*** ------------------------------------------------------------------

    DO 9 I=1,N5
      DO 8 J=1,N5
        A(I,J)=0.0D0
        B(I,J)=0.0D0
:     CONTINUE
)   CONTINUE
```

```
*  -----------------------------------------------------------------------
********************** THE ELEMENTS [ A11 & B11 ]
** -----------------------------------------------------------------------

      DO 11 I=3,N
        DO 10 J=1,N
          A(1,J)=1.0D0
          A(2,J)=(-1.0D0)**(J+1)
          A(I,J)=VNP(I-2,J)
          B(I,J)=XI(I-2,J)
         CONTINUE
      CONTINUE

** -----------------------------------------------------------------------
********************** THE ELEMENT [ A22 & B22 ]
** -----------------------------------------------------------------------

       DO 13 I=NL3,N2
         DO 12 J=NL1,N2
           A(NL1,J)=1.0D0
           A(NL2,J)=(-1.0D0)**(J-NL1)
           A(I,J)   =V(I-NL2,J-N)
           B(I,J)=  PRV* XI(I-NL2,J-N)
          CONTINUE
       CONTINUE

** ---------------------------------------------------------------------
********************** THE ELEMENT [ B33 ]
*** ---------------------------------------------------------------------

        DO 15 I=N2L3 , N3
         DO 14 J=N2L1 , N3
           B(I,J)=PRM*XI(I-N2L2,J-N2)
14        CONTINUE
15      CONTINUE

*** ----------------------------------------------------------------------
*********************** THE ELEMENT [ A33 ]
*** ----------------------------------------------------------------------

        DO 16 I=1,N
          A(N2L1,I+N2)= 2.0D0 * DBLE ((I-1)**2)
          A(N2L2,I+N2)= 2.0D0 * DBLE ((I-1)**2)*(-1.0D0)**(I)
6       CONTINUE

        DO 18 I=N2L3,N3
          DO 17 J=N2L1,N3
            A(I,J)= V(I-N2L2,J-N2)
7          CONTINUE
8       CONTINUE
```

```
**  ---------------------------------------------------------------------
************************ THE ELEMENTS [ A44 , B44 ]
**  ---------------------------------------------------------------------

      DO 19 I=1,N
        A(N3L1,I+N3)= 2.0D0 * DBLE ((I-1)**2)
        A(N3L2,I+N3)= (-1.0D0)**(I+1)
      CONTINUE

      DO 21 I=N3L3,N4
        DO 20 J=N3L1,N4
          A(I,J)= V(I-N3L2,J-N3)
          B(I,J)=PRS * XI(I-N3L2,J-N3)
        CONTINUE
      CONTINUE

**  ---------------------------------------------------------------------
************************** THE ELEMENT [ A55 ]
**  ---------------------------------------------------------------------

      DO 23 I=N4L3,N5
        DO 22 J=N4L1,N5
          A(N4L1,J)= 1.0D0
          A(N4L2,J)= (-1.0D0)**(J-N4L1)
          A(I,J)    = V(I-N4L2,J-N4)
2       CONTINUE
3     CONTINUE

***  --------------------------------------------------------------------
***************************  THE ELEMENT [A12]
***  --------------------------------------------------------------------

      DO 25 I=3,N
        DO 24 J=NL1,N2
          A(I,J)=(-1.0D0)* DSQRT(ST(K))*(A1**2)*XI(I-2,J-N)
4       CONTINUE
5     CONTINUE

***  --------------------------------------------------------------------
***************************  THE ELEMENT [A13]
***  --------------------------------------------------------------------

      DO 27 I=3,N
        DO 26 J=N2L1,N3
          A(I,J)=2.0D0*VD(I-2,J-N2)
26      CONTINUE
27    CONTINUE

***  --------------------------------------------------------------------
****************************** THE ELEMENT [ A14 ]
***  --------------------------------------------------------------------
```

```
      DO 29 I=3,N
        DO 28 J=N3L1,N4
          A(I,J)= DSQRT(SW(K))*(A1**2)*XI(I-2,J-N3)
        CONTINUE
      CONTINUE

**  -------------------------------------------------------------------
****************************  THE ELEMENT [ A25 ]
**  -------------------------------------------------------------------

      DO 31 I=NL3,N2
        DO 30 J=N4L1,N5
          A(I,J)=DSQRT(ST(K))*XI(I-NL2,J-N4)
        CONTINUE
      CONTINUE

**  -------------------------------------------------------------------
******************************** THE ELEMENT [A35]
**  -------------------------------------------------------------------

      DO 33 I=N2L3,N3
        DO 32 J=N4L1,N5
          A(I,J)=2.0D0*Q*D(I-N2L2,J-N4)
2       CONTINUE
3     CONTINUE

**  -------------------------------------------------------------------
*******************************  THE ELEMENT [ A45 ]
**  -------------------------------------------------------------------

      DO 35 I=N3L3,N4
        DO 34 J=N4L1,N5
          A(I,J)=(-1.0D0)*DSQRT(SW(K))*XI(I-N3L2,J-N4)
4       CONTINUE
5     CONTINUE

***  ------------------------------------------------------------------
********************************  THE ELEMENT [ A51 ]
***  ------------------------------------------------------------------

      DO 37 I=N4L3,N5
        DO 36 J=1,N
          A(I,J)=(-1.0D0)*XI(I-N4L2,J)
6       CONTINUE
7     CONTINUE

***  ------------------------------------------------------------------

***                        WE WILL USE NAGROUTINE (F02BJF)

      MATZ=.FALSE.
```

```
      EPS1=X02AAF(0.0D0)
      IFAIL=0

**  ----------------------------------------------------------------

      CALL F02BJF ( N5,A,L,B,L,EPS1,ALFR,ALFI,BETA,MATZ,Z,L,ITER,
 *                  IFAIL)
**  ----------------------------------------------------------------

      J=0
      DO 38 I=1,N5
        IF ( BETA(I).NE. 0.0D0) THEN
        J=1+J
        ELANDAR(J)=ALFR(I)/BETA(I)
        ELANDAI(J)=ALFI(I)/BETA(I)
        END IF
8     CONTINUE
      ELARGER=ELANDAR(1)
      DO 39 I=2,J
        IF ( ELANDAR(I).GT.ELARGER) THEN
         ELARGER=ELANDAR(I)
         ELARGEI=ELANDAI(I)
        END IF
9     CONTINUE
      GR(K)=ELARGER
      GI(K)=ELARGEI
      IF ( K.LT.2) GO TO 40
        IF(SS.EQ.0.0D0) THEN
          IF( DABS(ST(K)-ST(K-1)).LE.1.0D-7)THEN
            R=ST(K)
            SEGMAR=GR(K)
            SEGMAI=GI(K)
            RETURN
          END IF
          ELSE
          IF(DABS(SW(K)-SW(K-1)).LE.1.0D-7)THEN
            R=SW(K)
            SEGMAR=GR(K)
            SEGMAI=GI(K)
            RETURN
          END IF
        END IF
0   CONTINUE
    END

***  ---------------------------------------------------------------

*********** THIS SUBROUTINE CALCULATES THE PRODUCT OF TWO MATRICES

***  ---------------------------------------------------------------

    SUBROUTINE PRO (N,H,T,Z)
```

```
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      PARAMETER (L1=30)
      DIMENSION H(L1,L1), T(L1,L1), Z(L1,L1)
      DO 42 I=1,N
        DO 42 J=1,N
          Z(I,J)=0.0D0
          DO 41 K=1,N
            Z(I,J)=Z(I,J)+H(I,K)*T(K,J)
1         CONTINUE
2     CONTINUE
      RETURN
      END

*** ---------------------------------------------------------------

*************** THIS SUBROUTINE BUILDS THE DERIVATIVE MATRIX

*** ---------------------------------------------------------------

      SUBROUTINE DERIVE (N,D)
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      PARAMETER (L1=30)
      DIMENSION D(L1,L1)
      DO 43 I=1,N
        DO 43 J=1,N
          IF((J.GT.I) .AND. (MOD(J-I,2) .EQ. 1)) THEN
          D(I,J)=2.0D0*DBLE(J-1)
          ELSE
          D(I,J)=0.0D0
          END IF
3     CONTINUE
      DO 44 I=1,N
        D(1,I)=0.5D0*D(1,I)
4     CONTINUE
      RETURN
      END
```

APPENDIX 4

```
    PROGRAME RIGID 1
*** ----------------------------------------------------------------

*** THERMOSOLUTAL CONVECTION IN A HORIZONTAL POROUS LAYER AFFECTED
*** BY A VERTICAL MAGNETIC FIELD WHEN THE RELATION SHIP BETWEEN THE
*** MAGNETIC FIELD & THE MAGNETIC INDUCTION IS LINEAR, THE BOUNDARY
*** CONDITIONS ARE RIGID
**********   (w=theta=b=0), Dw=Dc=0     AT Z=0 , Z=1.      (CASE 1)
*** THE FLUID LAYER IS HEATED FROM BELOW & SOLUTED FROM ABOVE.
*** IN THIS PROGRAMME APPROXIMATION OF CHEBYSHEV POLYNOMIALS IS USED
*** TO SOLVE THE PROBLEM.

*** ---------------------------------------------------------------

************************* { MAIN PROGRAMME }

   IMPLICIT DOUBLE PRECISION (A-H,O-Z)
   PARAMETER  ( E = 1.0D-9 )
   COMMON N,R1,R2,PRV,PRS,PRM,NP,Q,SS,SSS
   OPEN (8,FILE='C:\NAGFL15D\BATCH\AAMAL\RESULT6')
   WRITE(*,*) ' ENTER N,A,B,R1,R2,PRV,PRS,PRM,NP,Q,SS,SSS'

*** ----------------------------------------------------------------

***** WHERE : (N)     IS THE NUMBER OF POLYNOMIALS.
*********** (A & B)    ARE TWO VALUES FOR THE WAVE NUMBER.
*********** (R1 & R2) ARE TWO VALUESFOR THE RAYLEIGH NUMBER.
*********** (PRV & PRS & PRM) BE RESPECTIVELY THE VISCOUS PRANDTLE
***************** NUMBER, THE SOLUTE PRANDTLE NUMBER AND THE MAGNETIC
***************** PRANDTLE NUMBER.
***********   (NP)    IS THE INVERSE OF THE NON-DIMENSIONAL PERMEABILITY.
***********   (Q)     IS CHANDRASEKHAR NUMBER.
***********   (SS)    IS THE PARAMETER WHICH DECIDES THE REQUIRD RAYLEIGH
***************** NUMBER THERMAL OR SOLUTE.
***********   (SSS)   IS THE VALUE OF THE KNOWN RAYLEIGH NUMBER.

*** -----------------------------------------------------------------

   READ*,N,A,B,R1,R2,PRV,PRS,PRM,NP,Q,SS,SSS
   RGOLD=(DSQRT(5.0D0)-1.0D0)/2.0D0
   M=DABS(DLOG(E/(B-A))/(DLOG(RGOLD)))
   A1=A+RGOLD*(B-A)

*** -----------------------------------------------------------------
   CALL CHEB(A1,U,G1R,G1I)
*** -----------------------------------------------------------------

   A2=A+RGOLD**2*(B-A)
```

```
**  ------------------------------------------------------------------
   CALL CHEB(A2,V,G2R,G2I)
**  ------------------------------------------------------------------

   DO 1 I=1,M
     IF (U.GT.V) THEN
      U=V
      B=A1
      A1=A2
      A2=A+RGOLD**2*(B-A)
***  -----------------------------------------------------------------
     CALL CHEB(A2,V,G2R,G2I)
***  -----------------------------------------------------------------

     ELSE
      V=U
      A=A2
      A2=A1
      A1=A+RGOLD*(B-A)
***  -----------------------------------------------------------------
     CALL CHEB(A1,U,G1R,G1I)
***  -----------------------------------------------------------------

     END IF
     IF ( DABS(U-V).LE.E) GO TO 50
1   CONTINUE
50  AMIN=(A1+A2)/2.0D0

***  -----------------------------------------------------------------
       CALL CHEB(AMIN,R,SEGMAR,SEGMAI)
***  -----------------------------------------------------------------

    PRINT*,'THE CRITICAL VALUE  OF THE  WAVE  NUMBER=',AMIN
    IF(SS.EQ.0.0D0) THEN
      PRINT*,'THE THERMAL RAYLEIGH NUMBER=',R
      PRINT*,'THE SOLUTE  RAYLEIGH NUMBER=',SSS
    ELSE
      PRINT*,'THE SOLUTE  RAYLEIGH NUMBER=',R
      PRINT*,'THE THERMAL RAYLEIGH NUMBER=',SSS
    END IF
    PRINT*,'THE REAL PART OF EIGENVALUE=',SEGMAR
    PRINT*,'THE IMAGINARY PART OF EIGENVALUE=',SEGMAI
    STOP
    END

****  ----------------------------------------------------------------
************ THIS SUBROUTINE IS USED TO EVALUATE THE RAYLEIGH NUMBER
****  ----------------------------------------------------------------
```

```
      SUBROUTINE CHEB(A1,R,SEGMAR,SEGMAI)
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      COMMON N,R1,R2,PRV,PRS,PRM,NP,Q,SS,SSS
      PARAMETER (L1=30, L=5*L1)
      DIMENSION A(L,L),B(L,L),XI(L1,L1),D(L1,L1),D2(L1,L1),V(L1,L1),
     *          VD(L1,L1),Z(L,L),ITER(L),ALFR(L),ALFI(L),BETA(L),GR(L),
     *          GI(L),ELANDAR(L),ELANDAI(L),ST(100),SW(100),VNP(L1,L1)
      LOGICAL   MATZ
      EXTERNAL  X02AAF
      EXTERNAL  F02BJF
      IF(SS.EQ.0.0D0)THEN
        ST(1)=R1
        ST(2)=R2
      ELSE
        SW(1)=R1
        SW(2)=R2
      END IF
      NL1=N+1
      NL2=N+2
      NL3=N+3
      N2=2*N
      N2L1=N2+1
      N2L2=N2+2
      N2L3=N2+3
      N3=3*N
      N3L1=N3+1
      N3L2=N3+2
      N3L3=N3+3
      N4=4*N
      N4L1=N4+1
      N4L2=N4+2
      N4L3=N4+3
      N5=5*N
*** ---------------------------------------------------------------
************************* THE IDENTITY MATRIX  (XI)
*** ---------------------------------------------------------------

      DO 3 I=1,N
        DO 2 J=1,N
          IF(I.EQ.J) THEN
           XI(I,J)= 1.0D0
          ELSE
           XI(I,J)=0.0D0
          END IF
 2      CONTINUE
 3    CONTINUE

*** ---------------------------------------------------------------
      CALL DERIVE (N,D)
      CALL PRO(N,D,D,D2)
*** ---------------------------------------------------------------
```

```
      DO 5 I=1,N
        DO 4 J=1,N
          V(I,J)=4.0D0*(D2(I,J)-((A1/2.0D0)**2*XI(I,J)))
        CONTINUE
*** ---------------------------------------------------------------
      CALL PRO (N,V,D,VD)
*** ---------------------------------------------------------------

      DO 7 I=1,N
        DO 6 J=1,N
          VNP(I,J)=V(I,J)-NP*XI(I,J)
        CONTINUE
      CONTINUE

*** ---------------------------------------------------------------
************************ THE SECANT METHOD
*** ---------------------------------------------------------------

      DO 42 K=1,100
        IF (SS.EQ.0.0D0) THEN
          SW(K)=SSS
          IF (K.GE.3) THEN
            ST(K)=(ST(K-2)*GR(K-1)-ST(K-1)*GR(K-2))/(GR(K-1)-GR(K-2))
          END IF
        ELSE
          ST(K)=SSS
          IF (K.GE.3) THEN
            SW(K)=(SW(K-2)*GR(K-1)-SW(K-1)*GR(K-2))/(GR(K-1)-GR(K-2))
          END IF
        END IF

*** ---------------------------------------------------------------
********************** THE A & B ZEROING MATRIX
*** ---------------------------------------------------------------

        DO 9 I=1,N5
          DO 8 J=1,N5
            A(I,J)=0.0D0
            B(I,J)=0.0D0
          CONTINUE
        CONTINUE

*** ---------------------------------------------------------------
******************** THE ELEMENTS [ A11 & B11 ]
*** ---------------------------------------------------------------

        DO 11 I=3,N
          DO 10 J=1,N
            A(I,J)=VNP(I-2,J)
            B(I,J)=XI(I-2,J)
```

```
0        CONTINUE
1     CONTINUE

*** ------------------------------------------------------------------
********************** THE ELEMENTS [ A22 & B22 ]
*** ------------------------------------------------------------------

      DO 13 I=NL3,N2
        DO 12 J=NL1,N2
          A(NL1,J)=1.0D0
          A(NL2,J)=(-1.0D0)**(J-NL1)
          A(I,J)   =V(I-NL2,J-N)
          B(I,J)= PRV* XI(I-NL2,J-N)
2       CONTINUE
3     CONTINUE

*** ------------------------------------------------------------------
********************** THE ELEMENTS [A33, B33 ]
*** ------------------------------------------------------------------

      DO 15 I=N2L3 , N3
        DO 14 J=N2L1 , N3
          A(N2L1,J)=1.0D0
          A(N2L2,J)=(-1.0D0)**(J-N2L1)
          A(I,J)= V(I-N2L2,J-N2)
          B(I,J)=PRM*XI(I-N2L2,J-N2)
14      CONTINUE
15    CONTINUE

*** ------------------------------------------------------------------
********************** THE ELEMENTS [ A44 , B44 ]
*** ------------------------------------------------------------------

      DO 16 I=1,N
        A(N3L1,I+N3)= 2.0D0 * DBLE ((I-1)**2)
        A(N3L2,I+N3)= 2.0D0*DBLE ((I-1)**2)*(-1.0D0)**(I)
.6    CONTINUE

      DO 18 I=N3L3,N4
        DO 17 J=N3L1,N4
          A(I,J)= V(I-N3L2,J-N3)
          B(I,J)=PRS * XI(I-N3L2,J-N3)
.7      CONTINUE
.8    CONTINUE

*** ------------------------------------------------------------------
************************* THE ELEMENT [ A55 ]
*** ------------------------------------------------------------------

      DO 20 I=N4L3,N5
        DO 19 J=N4L1,N5
          A(N4L1,J)= 1.0D0
```

```
        A(N4L2,J)=  (-1.0D0)**(J-N4L1)
        A(I,J)    = V(I-N4L2,J-N4)
)     CONTINUE
)   CONTINUE

*** ------------------------------------------------------------------
************************** THE ELEMENT [A12]
*** ------------------------------------------------------------------

    DO 22 I=3,N
      DO 21 J=NL1,N2
        A(I,J)=(-1.0D0)* DSQRT(ST(K))*(A1**2)*XI(I-2,J-N)
1     CONTINUE
2   CONTINUE

*** ------------------------------------------------------------------
************************** THE ELEMENT [B13]
*** ------------------------------------------------------------------

  DO 24 I=3,N
    DO 23 J=N2L1 , N3
        B(I,J)= (-1.0D0)* 2.0D0 * PRM * D(I-2,J-N2)
3    CONTINUE
4  CONTINUE

*** ------------------------------------------------------------------
************************** THE ELEMENT [ A14 ]
*** ------------------------------------------------------------------

    DO 28 I=3,N
      DO 27 J=N3L1,N4
        A(I,J)= DSQRT(SW(K))*(A1**2)*XI(I-2,J-N3)
7     CONTINUE
8   CONTINUE

*** ------------------------------------------------------------------
*************************** THE ELEMENT [A15]
*** ------------------------------------------------------------------

  DO 29 I=1,N
    A(1,I+N4)= 2.0D0*DBLE((I-1)**2)
    A(2,I+N4)= 2.0D0*DBLE((I-1)**2)*(-1.0D0)**(I)
9 CONTINUE

  DO 31 I=3,N
    DO 30 J=N4L1,N5
      A(I,J)= (-1.0D0) * 4.0D0 * Q * D2(I-2,J-N4)
0    CONTINUE
1  CONTINUE
```

```
**  ----------------------------------------------------------------------
***************************** THE ELEMENT [ A25 ]
**  ----------------------------------------------------------------------

      DO 33 I=NL3,N2
        DO 32 J=N4L1,N5
          A(I,J)=DSQRT(ST(K))*XI(I-NL2,J-N4)
2       CONTINUE
3     CONTINUE

**  ----------------------------------------------------------------------
****************************** THE ELEMENT [A35]
**  ----------------------------------------------------------------------

      DO 35 I=N2L3,N3
        DO 34 J=N4L1,N5
          A(I,J)=2.0D0*Q*D(I-N2L2,J-N4)
34      CONTINUE
35    CONTINUE

***  ---------------------------------------------------------------------
***************************** THE ELEMENT [ A45 ]
***  ---------------------------------------------------------------------

      DO 37 I=N3L3,N4
        DO 36 J=N4L1,N5
          A(I,J)=(-1.0D0)*DSQRT(SW(K))*XI(I-N3L2,J-N4)
6       CONTINUE
7     CONTINUE

***  ---------------------------------------------------------------------
***************************** THE ELEMENT [ A51 ]
***  ---------------------------------------------------------------------

      DO 39 I=N4L3,N5
        DO 38 J=1,N
          A(I,J)=(-1.0D0)*XI(I-N4L2,J)
8       CONTINUE
9     CONTINUE

***  ---------------------------------------------------------------------

                     WE WILL USE NAGROUTINE (F02BJF)

      MATZ=.FALSE.
      EPS1=X02AAF(0.0D0)
      IFAIL=0
```

```
**  ------------------------------------------------------------------

    CALL F02BJF ( N5,A,L,B,L,EPS1,ALFR,ALFI,BETA,MATZ,Z,L,ITER,
 *                IFAIL)

**  ------------------------------------------------------------------

    J=0
    DO 40 I=1,N5
      IF ( BETA(I).NE. 0.0D0) THEN
        J=1+J
        ELANDAR(J)=ALFR(I)/BETA(I)
        ELANDAI(J)=ALFI(I)/BETA(I)
      END IF
0   CONTINUE
    ELARGER=ELANDAR(1)
    DO 41 I=2,J
      IF ( ELANDAR(I).GT.ELARGER) THEN
      ELARGER=ELANDAR(I)
      ELARGEI=ELANDAI(I)
      END IF
1   CONTINUE
    GR(K)=ELARGER
    GI(K)=ELARGEI
    IF ( K.LT.2) GO TO 42
      IF(SS.EQ.0.0D0) THEN
        IF( DABS(ST(K)-ST(K-1)).LE.1.0D-7)THEN
          R=ST(K)
          SEGMAR=GR(K)
          SEGMAI=GI(K)
          RETURN
        END IF
        ELSE
        IF(DABS(SW(K)-SW(K-1)).LE.1.0D-7)THEN
          R=SW(K)
          SEGMAR=GR(K)
          SEGMAI=GI(K)
          RETURN
        END IF
      END IF
2  CONTINUE
   END

***  -----------------------------------------------------------------

*********** THIS SUBROUTINE CALCULATES THE PRODUCT OF TWO MATRICES

***  -----------------------------------------------------------------

   SUBROUTINE PRO (N,H,T,Z)
   IMPLICIT DOUBLE PRECISION (A-H,O-Z)
   PARAMETER (L1=30)
```

```
 DIMENSION H(L1,L1), T(L1,L1), Z(L1,L1)
 DO 44 I=1,N
   DO 44 J=1,N
     Z(I,J)=0.0D0
     DO 43 K=1,N
       Z(I,J)=Z(I,J)+H(I,K)*T(K,J)
      CONTINUE
  CONTINUE
  RETURN
  END
**  -----------------------------------------------------------------
************** THIS SUBROUTINE BUILDS THE DERIVATIVE MATRIX
**  -----------------------------------------------------------------
   SUBROUTINE DERIVE (N,D)
   IMPLICIT DOUBLE PRECISION (A-H,O-Z)
   PARAMETER (L1=30)
   DIMENSION D(L1,L1)
   DO 45 I=1,N
     DO 45 J=1,N
       IF((J.GT.I) .AND. (MOD(J-I,2) .EQ. 1)) THEN
       D(I,J)=2.0D0*DBLE(J-1)
       ELSE
       D(I,J)=0.0D0
       END IF
5  CONTINUE
   DO 46 I=1,N
     D(1,I)=0.5D0*D(1,I)
6  CONTINUE
   RETURN
   END
```

APPENDIX 5

```
      PROGRAME RIGID 2
***  -----------------------------------------------------------------------

***  THERMOSOLUTAL CONVECTION IN A HORIZONTAL POROUS LAYER AFFECTED
***  BY A VERTICAL MAGNETIC FIELD WHEN THE RELATION SHIP BETWEEN THE
***  MAGNETIC FIELD & THE MAGNETIC INDUCTION IS LINEAR, THE BOUNDARY
***  CONDITIONS ARE RIGID
************ (w=theta=b=0), Dw=0      AT Z=0 , Z=1.
************ C=0   AT  Z=0     &     DC=0     AT  Z=1.    (CASE 2)
***  THE FLUID LAYER IS HEATED FROM BELOW & SOLUTED FROM ABOVE.
***  IN THIS PROGRAMME APPROXIMATION OF CHEBYSHEV POLYNOMIALS IS USED
***  TO SOLVE THE PROBLEM.

***  ---------------------------------------------------------------------

********************** { MAIN PROGRAMME }

      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      PARAMETER  ( E = 1.0D-9 )
      COMMON N,R1,R2,PRV,PRS,PRM,NP,Q,SS,SSS
      OPEN (8,FILE='C:\NAGFL15D\BATCH\AAMAL\RESULT6')
      WRITE(*,*) ' ENTER N,A,B,R1,R2,PRV,PRS,PRM,NP,Q,SS,SSS'

***  ---------------------------------------------------------------------

***  WHERE : (N)     IS THE NUMBER OF POLYNOMIALS.
**********  A & B)    ARE TWO VALUES FOR THE WAVE NUMBER.
**********  (R1 & R2) ARE TWO VALUESFOR THE RAYLEIGH NUMBER.
**********  (PRV & PRS & PRM) BE RESPECTIVELY THE VISCOUS PRANDTLE
****************  NUMBER, THE SOLUTE PRANDTLE NUMBER AND THE MAGNETIC
****************  PRANDTLE NUMBER.
**********  (NP)    IS THE INVERSE OF THE NON-DIMENSIONAL PERMEABILITY.
**********  (Q)     IS CHANDRASEKHAR NUMBER.
**********  (SS)    IS THE PARAMETER WHICH DECIDES THE REQUIRD RAYLEIGH
****************  NUMBER THERMAL OR SOLUTE.
**********  (SSS)  IS THE VALUE OF THE KNOWN RAYLEIGH NUMBER.

***  ---------------------------------------------------------------------

      READ*,N,A,B,R1,R2,PRV,PRS,PRM,NP,Q,SS,SSS
      RGOLD=(DSQRT(5.0D0)-1.0D0)/2.0D0
      M=DABS(DLOG(E/(B-A))/(DLOG(RGOLD)))
      A1=A+RGOLD*(B-A)

**** ----------------------------------------------------------------------
      CALL CHEB(A1,U,G1R,G1I)
**** ----------------------------------------------------------------------
```

```
      A2=A+RGOLD**2*(B-A)
*  ----------------------------------------------------------------
      CALL CHEB(A2,V,G2R,G2I)
*  ----------------------------------------------------------------

      DO 1 I=1,M
        IF (U.GT.V) THEN
         U=V
         B=A1
         A1=A2
         A2=A+RGOLD**2*(B-A)
**  ---------------------------------------------------------------
       CALL CHEB(A2,V,G2R,G2I)
**  ---------------------------------------------------------------

        ELSE
         V=U
         A=A2
         A2=A1
         A1=A+RGOLD*(B-A)
**    -------------------------------------------------------------
        CALL CHEB(A1,U,G1R,G1I)
**    -------------------------------------------------------------

        END IF
        IF ( DABS(U-V).LE.E) GO TO 50
    1 CONTINUE
   50 AMIN=(A1+A2)/2.0D0
***  --------------------------------------------------------------
           CALL CHEB(AMIN,R,SEGMAR,SEGMAI)
***  --------------------------------------------------------------

      PRINT*,'THE CRITICAL VALUE  OF THE  WAVE  NUMBER=',AMIN
      IF(SS.EQ.0.0D0) THEN
        PRINT*,'THE THERMAL RAYLEIGH NUMBER=',R
        PRINT*,'THE SOLUTE  RAYLEIGH NUMBER=',SSS
      ELSE
        PRINT*,'THE SOLUTE  RAYLEIGH NUMBER=',R
        PRINT*,'THE THERMAL RAYLEIGH NUMBER=',SSS
      END IF
      PRINT*,'THE REAL PART OF EIGENVALUE=',SEGMAR
      PRINT*,'THE IMAGINARY PART OF EIGENVALUE=',SEGMAI
      STOP
      END
****  -------------------------------------------------------------
********** THIS SUBROUTINE IS USED TO EVALUATE THE RAYLEIGH NUMBER
****  -------------------------------------------------------------
```

```
      SUBROUTINE CHEB(A1,R,SEGMAR,SEGMAI)
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      COMMON N,R1,R2,PRV,PRS,PRM,NP,Q,SS,SSS
      PARAMETER (L1=30, L=5*L1)
      DIMENSION A(L,L),B(L,L),XI(L1,L1),D(L1,L1),D2(L1,L1),V(L1,L1),
     *          VD(L1,L1),Z(L,L),ITER(L),ALFR(L),ALFI(L),BETA(L),GR(L),
     *          GI(L),ELANDAR(L),ELANDAI(L),ST(100),SW(100),VNP(L1,L1)
      LOGICAL   MATZ
      EXTERNAL  X02AAF
      EXTERNAL  F02BJF
      IF(SS.EQ.0.0D0)THEN
        ST(1)=R1
        ST(2)=R2
      ELSE
        SW(1)=R1
        SW(2)=R2
      END IF
      NL1=N+1
      NL2=N+2
      NL3=N+3
      N2=2*N
      N2L1=N2+1
      N2L2=N2+2
      N2L3=N2+3
      N3=3*N
      N3L1=N3+1
      N3L2=N3+2
      N3L3=N3+3
      N4=4*N
      N4L1=N4+1
      N4L2=N4+2
      N4L3=N4+3
      N5=5*N

*** -----------------------------------------------------------------
************************ THE IDENTITY MATRIX  (XI)
*** -----------------------------------------------------------------

      DO 3 I=1,N
        DO 2 J=1,N
          IF(I.EQ.J) THEN
           XI(I,J)= 1.0D0
          ELSE
           XI(I,J)=0.0D0
          END IF
2       CONTINUE
3     CONTINUE
```

```
*** ----------------------------------------------------------------
      CALL DERIVE (N,D)
      CALL PRO(N,D,D,D2)
*** ----------------------------------------------------------------

      DO 5 I=1,N
        DO 4 J=1,N
          V(I,J)=4.0D0*(D2(I,J)-((A1/2.0D0)**2*XI(I,J)))
        CONTINUE

*** ----------------------------------------------------------------
      CALL PRO (N,V,D,VD)
*** ----------------------------------------------------------------

      DO 7 I=1,N
        DO 6 J=1,N
          VNP(I,J)=V(I,J)-NP*XI(I,J)
        CONTINUE
      CONTINUE

*** ----------------------------------------------------------------
*********************** THE SECANT METHOD
*** ----------------------------------------------------------------

      DO 42 K=1,100
        IF (SS.EQ.0.0D0) THEN
          SW(K)=SSS
          IF (K.GE.3) THEN
            ST(K)=(ST(K-2)*GR(K-1)-ST(K-1)*GR(K-2))/(GR(K-1)-GR(K-2))
          END IF
        ELSE
          ST(K)=SSS
          IF (K.GE.3) THEN
            SW(K)=(SW(K-2)*GR(K-1)-SW(K-1)*GR(K-2))/(GR(K-1)-GR(K-2))
          END IF
        END IF

*** ----------------------------------------------------------------
************************** THE A & B ZEROING MATRIX
*** ----------------------------------------------------------------

        DO 9 I=1,N5
          DO 8 J=1,N5
            A(I,J)=0.0D0
            B(I,J)=0.0D0
8         CONTINUE
9       CONTINUE

*** ----------------------------------------------------------------
************************** THE ELEMENTS [ A11 & B11 ]
*** ----------------------------------------------------------------
```

```
      DO 11 I=3,N
        DO 10 J=1,N
          A(I,J)=VNP(I-2,J)
          B(I,J)=XI(I-2,J)
         CONTINUE
      CONTINUE

**  -----------------------------------------------------------------
************************ THE ELEMENTS [ A22 & B22 ]
**  -----------------------------------------------------------------

      DO 13 I=NL3,N2
        DO 12 J=NL1,N2
          A(NL1,J)=1.0D0
          A(NL2,J)=(-1.0D0)**(J-NL1)
          A(I,J)   =V(I-NL2,J-N)
          B(I,J)= PRV* XI(I-NL2,J-N)
2        CONTINUE
3      CONTINUE

***  ---------------------------------------------------------------
************************ THE ELEMENTS [A33, B33 ]
***  ---------------------------------------------------------------

      DO 15 I=N2L3 , N3
        DO 14 J=N2L1 , N3
          A(N2L1,J)=1.0D0
          A(N2L2,J)=(-1.0D0)**(J-N2L1)
          A(I,J)= V(I-N2L2,J-N2)
          B(I,J)=PRM*XI(I-N2L2,J-N2)
14       CONTINUE
15     CONTINUE

***  ---------------------------------------------------------------
************************ THE ELEMENTS [ A44 , B44 ]
***  ---------------------------------------------------------------

      DO 16 I=1,N
        A(N3L1,I+N3)= 2.0D0 * DBLE ((I-1)**2)
        A(N3L2,I+N3)= (-1.0D0)**(I+1)
6     CONTINUE

      DO 18 I=N3L3,N4
        DO 17 J=N3L1,N4
          A(I,J)= V(I-N3L2,J-N3)
          B(I,J)=PRS * XI(I-N3L2,J-N3)
7        CONTINUE
8      CONTINUE

****  --------------------------------------------------------------
************************* THE ELEMENT [ A55 ]
****  --------------------------------------------------------------
```

```
      DO 20 I=N4L3,N5
        DO 19 J=N4L1,N5
          A(N4L1,J)= 1.0D0
          A(N4L2,J)= (-1.0D0)**(J-N4L1)
          A(I,J)    = V(I-N4L2,J-N4)
9       CONTINUE
0     CONTINUE

*** ------------------------------------------------------------------
************************** THE ELEMENT [A12]
*** ------------------------------------------------------------------

      DO 22 I=3,N
        DO 21 J=NL1,N2
          A(I,J)=(-1.0D0)* DSQRT(ST(K))*(A1**2)*XI(I-2,J-N)
1       CONTINUE
2     CONTINUE

*** ------------------------------------------------------------------
************************** THE ELEMENT [B13]
*** ------------------------------------------------------------------

    DO 24 I=3,N
      DO 23 J=N2L1 , N3
          B(I,J)= (-1.0D0)* 2.0D0 * PRM * D(I-2,J-N2)
3     CONTINUE
4   CONTINUE

*** ------------------------------------------------------------------
************************** THE ELEMENT [ A14 ]
*** ------------------------------------------------------------------

      DO 28 I=3,N
        DO 27 J=N3L1,N4
          A(I,J)= DSQRT(SW(K))*(A1**2)*XI(I-2,J-N3)
7       CONTINUE
8     CONTINUE

*** ------------------------------------------------------------------
************************** THE ELEMENT [A15]
*** ------------------------------------------------------------------

    DO 29 I=1,N
      A(1,I+N4)= 2.0D0*DBLE((I-1)**2)
      A(2,I+N4)= 2.0D0*DBLE((I-1)**2)*(-1.0D0)**(I)
29  CONTINUE

    DO 31 I=3,N
      DO 30 J=N4L1,N5
        A(I,J)= (-1.0D0) * 4.0D0 * Q * D2(I-2,J-N4)
30    CONTINUE
```

```
. CONTINUE

:** -----------------------------------------------------------------
:************************** THE ELEMENT [ A25 ]
:** -----------------------------------------------------------------

      DO 33 I=NL3,N2
        DO 32 J=N4L1,N5
          A(I,J)=DSQRT(ST(K))*XI(I-NL2,J-N4)
2       CONTINUE
3     CONTINUE

*** -----------------------------------------------------------------
***************************** THE ELEMENT [A35]
*** -----------------------------------------------------------------

      DO 35 I=N2L3,N3
        DO 34 J=N4L1,N5
          A(I,J)=2.0D0*Q*D(I-N2L2,J-N4)
34      CONTINUE
35    CONTINUE

*** -----------------------------------------------------------------
****************************** THE ELEMENT [ A45 ]
*** -----------------------------------------------------------------

      DO 37 I=N3L3,N4
        DO 36 J=N4L1,N5
          A(I,J)=(-1.0D0)*DSQRT(SW(K))*XI(I-N3L2,J-N4)
6       CONTINUE
7     CONTINUE

*** -----------------------------------------------------------------
****************************** THE ELEMENT [ A51 ]
*** -----------------------------------------------------------------

      DO 39 I=N4L3,N5
        DO 38 J=1,N
          A(I,J)=(-1.0D0)*XI(I-N4L2,J)
8       CONTINUE
9     CONTINUE

*** -----------------------------------------------------------------

************************** WE WILL USE NAGROUTINE (F02BJF)

      MATZ=.FALSE.
      EPS1=X02AAF(0.0D0)
      IFAIL=0
```

```
** -------------------------------------------------------------------
      CALL F02BJF ( N5,A,L,B,L,EPS1,ALFR,ALFI,BETA,MATZ,Z,L,ITER,
 *                  IFAIL)
** -------------------------------------------------------------------

      J=0
      DO 40 I=1,N5
        IF ( BETA(I).NE. 0.0D0) THEN
          J=1+J
          ELANDAR(J)=ALFR(I)/BETA(I)
          ELANDAI(J)=ALFI(I)/BETA(I)
        END IF
      CONTINUE
      ELARGER=ELANDAR(1)
      DO 41 I=2,J
        IF ( ELANDAR(I).GT.ELARGER) THEN
        ELARGER=ELANDAR(I)
        ELARGEI=ELANDAI(I)
        END IF
      CONTINUE
      GR(K)=ELARGER
      GI(K)=ELARGEI
      IF ( K.LT.2) GO TO 42
        IF(SS.EQ.0.0D0) THEN
          IF( DABS(ST(K)-ST(K-1)).LE.1.0D-7)THEN
            R=ST(K)
            SEGMAR=GR(K)
            SEGMAI=GI(K)
            RETURN
          END IF
          ELSE
          IF(DABS(SW(K)-SW(K-1)).LE.1.0D-7)THEN
            R=SW(K)
            SEGMAR=GR(K)
            SEGMAI=GI(K)
            RETURN
          END IF
        END IF
2   CONTINUE
    END
*** -------------------------------------------------------------------

*********** THIS SUBROUTINE CALCULATES THE PRODUCT OF TWO MATRICES

*** -------------------------------------------------------------------

      SUBROUTINE PRO (N,H,T,Z)
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      PARAMETER (L1=30)
      DIMENSION H(L1,L1), T(L1,L1), Z(L1,L1)
      DO 44 I=1,N
```

```
   DO 44 J=1,N
     Z(I,J)=0.0D0
     DO 43 K=1,N
       Z(I,J)=Z(I,J)+H(I,K)*T(K,J)
     CONTINUE
: CONTINUE
  RETURN
  END
**  -----------------------------------------------------------------
**************** HIS SUBROUTINE BUILDS THE DERIVATIVE MATRIX
**  -----------------------------------------------------------------
  SUBROUTINE DERIVE (N,D)
  IMPLICIT DOUBLE PRECISION (A-H,O-Z)
  PARAMETER (L1=30)
  DIMENSION D(L1,L1)
  DO 45 I=1,N
    DO 45 J=1,N
      IF((J.GT.I) .AND. (MOD(J-I,2) .EQ. 1)) THEN
      D(I,J)=2.0D0*DBLE(J-1)
      ELSE
      D(I,J)=0.0D0
      END IF
5  CONTINUE
   DO 46 I=1,N
     D(1,I)=0.5D0*D(1,I)
6  CONTINUE
   RETURN
   END
```

وزارة التعليم العالي
جامعة أم القرى
كلية العلوم التطبيقية

بسم الله الرحمن الرحيم

محضر مناقشة رسالة (ماجستير)

الحمد لله والصلاة والسلام على رسول الله صلى الله عليه وسلم وعلى آله وصحبه ومن والاه ففي تمام الساعة ...١٠... من يوم ...الاثنين... الموافق ١٩/٢/١٤٢٤هـ... اجتمعت اللجنة المشكلة بقرار مجلس كلية رقم ...٤...... وفي جلسته ...٤... بتاريخ ٢٨/١/١٤٢٤هـ والمكونة من أصحاب السعادة :

م	الاسم	الجهة التابع لها	وضع المناقش في اللجنة
١	د. محمد عبد الجليل جبارة	كلية المعلمين بمكة	مناقش خارجي
٢	د. عبد الفتاح بن قاري عبد القادر بخاري	جامعة أم القرى	مناقش داخلي
٣	أ.د. عبد الله أحمد عبد الله	جامعة أم القرى	مقرر الجلسة ومشرف

لمناقشة الرسالة المقدمة لنيل درجة الماجستير في العلوم الرياضية ..
من الطالب / الطالبة ..آمال بن سعيد حاتم..........................
تخصص ...رياضيات تطبيقية...... بكلية العلوم التطبيقية بمكة المكرمة بجامعة أم القرى .
بعنوان ...انتقال الذبذبات الحرارية لطبقة مسامية افقية تحت تأثير مجال مغناطيسي عمودي..........
وبعد الانتهاء اللجنة من المناقشة في الساعة ...١١...... والتداول فيما بينها .
وبناء على موقف الطالب / الطالبة أثناء المناقشة .
أوصت اللجنة بمنح درجة الماجستير في العلوم الرياضية بتقدير ...ممتاز..... بالدرجة ...٩١,٤٠٥..... الموضحة . والله الموفق .
ما اكتسبه من تقديرات بالسنة المنهجية .....٨٥,٥.... بنسبة ٢٥,٦٥ / ٣٠
ما اكتسبه من الساعات المعتمدة المطلوبة لهذه الدرجة ....٢٤........

ملاحظات

....
....

م	الاسم	الدرجة	التوقيع
١	د. محمد عبد الجليل جبارة	٩٢	(توقيع)
٢	د. عبد الفتاح بن قاري عبد القادر بخاري	٩٤	(توقيع)
٣	أ.د. عبد الله أحمد عبد الله .	٩٥	(توقيع)
	مجموع الدرجات	٢٨٢	
	المتوسط	٩٤	

رئيس قسم العلوم الرياضية

د . عبدالفتاح بن قاري بخاري

جامعة أم القرى - كلية العلوم التطبيقية - قسم العلوم الرياضية - صورة طبق الأصل

شرف...

بسم الله الرحمن الرحيم

المملكة العربية السعودية
وزارة التعليم العالي
جامعة أم القرى
كلية العلوم التطبيقية
قسم العلوم الرياضية

إنتقال الذوبان الحراري لطبقة مسامية أفقية تحت تأثير مجال مغناطيسي عمودي

اعداد
آمال بنت سعيد أحمد حاتم

اشراف
أ. د. عبدالله بن أحمد عبدالله

بحث تكميلي لمتطلبات الحصول على درجة الماجستير في الرياضيات التطبيقية

١٤٢٣ هـ - ٢٠٠٢ م

الملخص

انتقال الذوبان الحراري لطبقة أفقية مسامية تحت تأثير مجال مغناطيسي عمودي

يهدف هذا البحث الى دراسة عدم الاستقرار الحراري لمسألة بينارد لطبقة أفقية مسامية لانهائية يتخللها مائع تتخلله مادة مذابة، وهذا المائع لزج وموصل كهربيا وحراريا وغير قابل للانضغاط ويتعرض لتأثير مجال مغناطيسي عمودي ثابت .

ويتكون البحث من ثلاثة فصول رئيسية :

الفصل الاول : المقدمة

يحتوي هذا الفصل على مجموعة من الدراسات والأبحاث السابقة حول مسألة بينارد وقد تم تصنيفها الى أربعة محاور :

المحور الأول : يدور حول الدراسات السابقة عن مسألة بينارد بصورة عامة .

المحور الثاني : يدور حول الدراسات السابقة عن مسألة بينارد في حالة وجود وسط مسامي.

المحور الثالث : يدور حول الدراسات السابقة عن مسألة بينارد عندما يتخلل المائع مادة مذابة.

المحور الرابع : يدور حول الدراسات السابقة عن مسألة بينارد في حالة وجود وسط مسامي تتخلله مادة مذابة .

الفصل الثاني : طريقة شيبيشيف لحل مسائل القيمة الحدية

يتحدث هذا الفصل عن كثيرات حدود شيبيشيف وعن أهم خواصها حيث أنها تعتبر من أدق الطرق المستخدمة لحل مسائل القيمة الحدية، ويتضمن الفصل شرحا مفصلا لهذه الطريقة باستخدام أحد الأمثلة . كما يتضمن شرحا لطريقة (Golden section search) والتي تستخدم لايجاد النهاية الصغرى لعدد ريلي (Rayleigh number) .

الفصل الثالث : انتقال الذوبان الحراري لطبقة أفقية مسامية تحت تأثير مجال مغناطيسي عمودي .

يتضمن هذا الفصل موضوع البحث وكيفية الحصول على المعادلات الأساسية التي تحكم المسألة مع شرح للشروط الحدية . وقد تم مناقشة عدم الاستقرار لهذه المسألة من الناحية التحليلية في الحالات التالية :

١- عند تسخين المائع من الأعلى وزيادة تركيز المذاب من الأسفل .
٢- عند تسخين المائع من الأعلى وزيادة تركيز المذاب من الأعلى .
٣- عند تسخين المائع من الأسفل وزيادة تركيز المذاب من الأعلى .
٤- عند تسخين المائع من الأسفل وزيادة تركيز المذاب من الأسفل .

وايجاد الحل العددي للمسألة باستخدام طريقة شيبيشيف العددية ، وتم عرض بعض النتائج العددية عندما يكون السطحان الحديان من النوع الحر أو الجاسئ لحالتين مختلفتين للشرطين الحديين الخاصين بالمذاب .

<u>ومن أهم النتائج التي تم التوصل اليها :</u>

- لايمكن أن تحدث حالة عدم الاستقرار الثابت (Stationary instability) عند تسخين المائع من الأعلى وزيادة تركيز المذاب من الأسفل ، ويمكن حدوث هذه الحالة تحت شروط معينة في الحالات الثلاث الباقية لوضعي التسخين وزيادة تركيز المادة المذابة .

- يمكن أن تحدث حالة عدم الاستقرار المفرط (Overstability) في جميع الحالات الأربعة لوضعي التسخين وزيادة تركيز المادة المذابة ولكن تحت شروط معينة .

- تأثير المادة المذابة على استقرار المائع يعتمد على وضعي التسخين وزيادة تركيز المادة المذابة ، فاذا كان تركيزها يتزايد من الجانب السفلي لطبقة المائع فسيساعد وجودها على زيادة استقرار المائع بينما لايكون كذلك اذا كان تركيزها يتزايد من الجانب العلوي وذلك في حالة تسخين طبقة المائع من الجانب السفلي ، أما في حالة تسخين طبقة المائع من الجانب العلوي فسيكون العكس تماما .

- تأثير شدة المجال المغناطيسي على استقرار المائع يعتمد على وضعية التسخين فقط ، حيث يكون له تأثير مستقر عندما يكون التسخين من الاسفل ، بينما يكون العكس عندما يكون التسخين من الجانب العلوي .

- ان الوسط المسامي يعمل على زيادة ثبات المائع المغناطيسي .

- تأثير نفاذية الوسط المسامي على استقرار المائع يعتمد على وضعية التسخين فقط ، حيث يكون له تأثير مستقر عندما يكون التسخين من الاسفل ، بينما يكون العكس عندما يكون التسخين من الجانب العلوي .

- ان العلاقة بين عدد ريلي الخاص بالحرارة R_t وعدد ريلي الخاص بالمذاب R_s هي علاقة عكسية بينما تكون العلاقة بين عدد شاندرازكر Q وبين R_t ، R_s ، a هي علاقة طردية سواء كان السطحان الحديان من النوع الحر أو النوع الجاسئ .

SUMMARY

Thermosolutal convection in a horizontal porous layer affected by a vertical magnetic field.

This thesis examines the thermosolutal convection of an infinite horizontal layer occupied by a porous medium permeated by an incompressible thermally and electrically conducting viscous solutal fluid in the presence of a uniform vertical magnetic field. The problem is discussed for the following cases:

1- The fluid is heated from above and the solute concentration decreases upwards.
2- The fluid is heated from above and the solute concentration increases upwards.
3- The fluid is heated from below and the solute concentration increases upwards.
4- The fluid is heated from below and the solute concentration decreases upwards.

It has been shown that the stationary convection is not possible when the fluid is heated from above and soluted from below. However it can be possible in the other three cases under certain conditions. On the other hand overstability case is possible in all the four cases provided certain conditions are satisfied.

The effects of solute concentration, magnetic field and porosity on the system have been discussed. Numerical results are obtained using expansions in Chebyshev polynomials for the case when the fluid is heated from below and the solute concentration increases upwards for different boundary conditions.